Decision Making When Things Are Only a Matter of Time

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Abstract

This paper gives a comprehensive treatment of delay risk preferences, i.e., preferences towards the risk of something happening sooner or later. My results can be used to identify old and new curvature properties of the discount function, and they explain how outcomes that occur at unsure times ought to be discounted. I introduce and characterize the notions of *prudent* and *temperate* discounting in analogy to utility theory, and study their behavioral implications. The importance of delay risk preferences is illustrated through several applications in which certain outcomes are "only a matter of time," including investment, saving, and climate change.

Keywords: risk preferences; time preferences; prudence; precautionary behavior; investment

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a matter (also question) of time: "said with reference to an event or circumstance that is thought certain to come about, or to resolve itself in a particular way, sooner or later." From: *Oxford English Dictionary*[1]

1 Introduction

When researchers speak of decision making under risk, they usually refer to the risk of *what* may happen. The outcome at risk may be monetary or concern one's health; it may affect current or future well-being; and it may concern oneself and/ or others. In standard economic theory, the preferences towards such outcome risks are determined by a utility function. This paper is concerned with a fundamentally different type of risk which is about *when* something may happen. While most articles on risk assume perfect knowledge about when outcomes materialize and that uncertainty is restricted to how good or bad the outcome is gonna be, this paper studies the dual case where the outcome is known but the time of its occurrence is not. Everyday language has an expression for outcomes that are certain but whose time of occurrence is not. Such outcomes are said to be "only a matter of time."

Examples of outcomes that are only a matter of time, i.e., that are certain to come about sooner or later, are abundant. Moreover, the risk about when something will happen can be quite significant. While great research may be recognized eventually, for a tenure-tracker there is substantial risk that it will happen a few years too late. A similar risk is born by artists, musicians, or writers, being that their creations are not recognized during their lifetimes – or too late therein to enjoy the benefits. A saying says that time heals every broken heart. If it is true, the only question is whether this will happen sooner or later. It is also only a matter of time until climate change will cause a rise in the sea level, high enough to make parts of land uninhabitable. Given a genetic disposition (e.g., HIV), the development of an illness (AIDS) may be only a matter of time. Death is only a matter of time: "Mors certa, hora incerta" (death is certain, the hour is not) is engraved on the clock of the New Town Hall in Leipzig, Germany. Accordingly, the amount to be received from a standard life insurance is certain, but the time of receipt is not. Another example from economics is central bank policy. At times it is thought certain that interest rates will increase, but the important question is whether this will happen sooner or later. This paper analyzes risk preferences and behavior in such situations, situations where things are only a matter of time.

To become a bit more formal, compare the following two risks where all uncertainty is resolved immediately after the choice is made:

(i) With probability 50% receive $ 5,000 in 1 year and with probability 50% receive $ 10,000

[1] "time, n., int., and conj. Phrase P1 d." OED Online. Oxford University Press, March 2016. Web. 11 April 2016. The hyperlink requires access to OED Online.

in 1 year.

(ii) With probability 50% receive \$ 5,000 in 1 year and with probability 50% receive \$ 5,000 in 3 years.

Note that risk (i) is exclusively about what may happen which is why we call it a *(pure, i.e., delay-certain) outcome risk*. Risk (ii) does not concern what may happen but only when it may happen, and we call it a *(pure, i.e., outcome-certain) delay, or time, risk*.[2] The bulk of the literature has focused on pure outcome risks and thereby abstracted from delay risk. As a first step, this paper takes the other extreme and focuses on pure delay risks, which precisely describe situations where things are only a matter of time. The aim of this paper is to understand risk preferences towards delay risk (for example, would you prefer \$5000 in 2 years for sure over the time risk (ii) above?), as well as to analyze behavior in situations where things are only a matter of time.

Obviously, many interesting applications are a matter of both what may happen and when it may happen, i.e., a matter of both outcome and time. For example, the lender of a defaulted loan cares about both how much is repaid *and* about when it is repaid. In this paper, however, we focus on situations where things are *only* a matter of time, because this will allow us to study time risk preferences in isolation, i.e., independently from outcome preferences. We will investigate, though, to what extent results generalize to settings where both time and outcome are risky. It is easily seen that all results in this paper readily generalize to risky outcomes (so that things are *also* – but not only – a matter of time) as long as outcome risk is independent from delay risk. Once this independence assumption is relaxed, choices generally depend on both delay and outcome risk preferences in complicated ways. The results from this paper, however, will remain important even in that case. The same is true for results on pure outcome risk preferences – by no means does the literature on "usual" outcome risk preferences become useless once taking into account the neglected aspect of delay risk.

The starting point and key insight of this paper is that, within the seminal discounted expected utility (DEU) model of Ramsey (1928), Samuelson (1937), and Koopmans (1960), delay risk preferences are determined by the *expected discount (factor)* that applies to the outcome which is only a matter of time. In particular, delay risk preferences are determined by the discount function alone. This result is notable from an empirical viewpoint as it shows that choices over pure delay risks can be used to elicit individuals' discount functions independently from utility. Moreover, "expected discount" is mathematically similar to (atemporal) "expected utility." Through a result that I call the *time-outcome risk duality* one can thus transfer known results about utility functions

[2] To avoid confusion, note that a delay risk pays exactly once (so that the timing of the payment is uncertain), which is fundamentally different from several outcome risks which may or may not pay in each of several periods. That is, risk (ii) above is not the same as an outcome that pays \$0 or \$ 5000 with equal probability, both in year 1 and in year 3.

to obtain yet unknown results about discount functions. While this relationship may seem obvious ex-post, it has neither been formalized yet nor has it been systematically exploited. – The necessary tool to advance the analysis is the *discount premium,* which I define in analogy to the utility premium of Friedman and Savage (1948). The discount premium measures the change in the discount factor due to making the outcome's time of receipt risky. Because discount functions have a different shape than utility functions, however, the study of time risk preferences turns out to be quite intricate.

Let us first assume that the outcome whose delay risk we study is desirable. A first result is that convex discounting implies risk-seeking over time risk, an important insight made earlier by Onay and Öncüler (2007) and DeJarnette et al. (2015), who give an elegant and direct proof. I show that, with the time-outcome risk duality at hand, the result also follows from Rothschild and Stiglitz' (1970) seminal article on mean-preserving spreads. Moreover, the time-outcome risk duality allows us to go further. I continue the study of time risk preferences by characterizing comparative time risk-seeking in the spirit of de Finetti (1952), Arrow (1965), and Pratt (1964). A DM is more willing to wait for a delay risk than another DM if and only if her discount function's Arrow-Pratt coefficient is larger. Another result shows that investment is more attractive if the uncertainty in the time of payback increases.

It has been well understood by now that the higher-order risk preferences of *prudence* ($u''' \geq 0$ in EU, Kimball 1990) and *temperance* ($u'''' \leq 0$, Kimball 1993) complement risk aversion in important ways; for an overview consult Gollier (2001). For example, risk aversion is neither necessary nor sufficient for downside risk aversion (Menezes et al., 1980) or precautionary saving (Leland, 1968; Sandmo, 1970), but prudence is. Temperance is decisive for behavior in the presence of background risk (Eeckhoudt et al., 1996). For example, temperance determines saving behavior when taking into account macroeconomic risks such as interest rate risk (Eeckhoudt and Schlesinger, 2008).

I define the higher-order time risk preferences of *prudent discounting* and *intemperate discounting* via preferences over simple 50-50 *time risk apportionment* lotteries in the spirit of Eeckhoudt and Schlesinger (2006). Intuitively, prudent discounting indicates a preference for facing an unavoidable zero-mean time risk in the state where delay is shorter. Intemperate discounting means that independent zero-mean delay risks are mutually enticing, i.e., their joint occurrence in a state is preferred over occurrence in two separate states. Using the discount premium, I show that these lottery preferences are necessary and sufficient for a negative third[3] and positive fourth derivative of the discount function, respectively. I show that discount prudence is necessary and sufficient for a *precautionary waiting motive.* A DM is strictly willing to delay a desirable outcome if in exchange he receives an identical outcome that occurs at a later but uncertain time earlier. I show

[3]The change in sign for prudence is a consequence of the discount function being decreasing.

that discount prudence is the property that characterizes downside risk aversion and skewness preference towards delay risk, and that discount intemperance implies a preference for fat-tailed delay risks.

All "commonly used utility functions" (Brockett and Golden, 1987)[title] are increasing with derivatives that alternate in sign, a property called *mixed risk aversion* by (Caballé and Pomansky, 1996). Mixed risk aversion has been shown to have far-reaching consequences for the risk-taking behavior in expected utility. Having characterized the signs of the discount functions' derivatives through behavioral preferences over simple lotteries, it is interesting to note (and easily verified) that the commonly used discount functions are *de*creasing with derivatives that alternate in sign. Since this is directly opposite to the utility case, I refer to the commonly used discount functions as being *anti-mixed (risk-averse).* Therefore, all commonly used discount functions are prudent and intemperate, which highlights the need to understand these concepts. I proceed by giving a behavioral characterization of anti-mixed discounting. Anti-mixed time risk preferences follow a simple intuition: First, "less delay is better". Second, it is desirable to consistently gather unavoidable good things (such as delay reductions or zero-mean time risks) in the same state of nature rather than distributing them across two different states. This consistency heuristic can be referred to as a *preference for combining "good" with "good"*; cf. Crainich et al. (2013). Ebert et al. (2016) show that a discount function can be represented as a group discount function (e.g., Jackson and Yariv 2014) if and only if it is anti-mixed. The current paper explains what anti-mixedness means for time risk preferences, i.e., it also explains the time risk preferences of groups.

To keep the confusion to a minimum, I study the equally important case of delay risk preferences towards *un*desirable outcomes in a separate section. By then, it will be clear to the reader that delay risk preferences towards undesirable outcomes are "like risk-averse outcome risk preferences." Among other results, I obtain a new characterization of exponential discounting. An individual is an exponential discounter if and only if her delay risk preferences towards the arrival of an undesirable outcome exhibit constant absolute risk aversion (CARA). If π_d denotes the number of days by which a decision maker is willing to face a catastrophe earlier in exchange for facing it at a certain rather than at an uncertain time, then, under CARA towards time risk, π_d is independent of the catastrophe's expected time of occurrence.

For general anti-mixed discount functions, the uncertainty about when undesirable effects come into effect are utility-decreasing. I show that the willingness to pay for the abatement of climate change increases with the associated time risk. I provide a generalization of the net present value (NPV) rule for evaluating investment opportunities that accounts for time risk. In particular, the value of investing into abatement must be adjusted upward when taking into account any sort of time risk.

This paper contributes to the large literature on risk and time preferences by studying the neglected class of time risks. In particular, to the best of my knowledge there are only three papers that are concerned with time risk in some way. Chesson and Viscusi (2003) conduct a survey on business owners and find that 30% of them exhibit "lottery timing risk aversion". Moreover, the authors find that aversion to uncertain timing is related to an aversion to ambiguous probabilities involving storm damage risks. Onay and Öncüler (2007) conduct three surveys and provide additional evidence for time risk aversion. Moreover, they show that, within DEU, a convex discount function is inconsistent with weak time risk aversion (i.e., the expected value of any lottery is preferred over the lottery itself). They further show that their data can be explained with probability weighting. Finally, DeJarnette et al. (2015) are the first to provide incentivized experimental evidence on time risk aversion in a large sample of 197 subjects. They further provide an insightful theoretical analysis that shows that Epstein-Zin preferences can explain their findings, and even better so than probability weighting.

My paper has the different objective of providing a characterization of time risk preferences within the normatively important DEU model. My main idea is to clarify and formalize the similarity between delay and univariate outcome risk preferences by considering situations where things are only a matter of time. This similarity is subsequently exploited to study risk-taking behavior which goes beyond that determined by the convexity of the discount function alone. I show that other curvature properties of the discount function (e.g., the Arrow-Pratt measure, prudence, anti-mixedness) are important as well. Unlike the above papers, I study the behavioral implications of time risk preferences in a number of applied settings. This requires the study of time risks which are not necessarily binary and/ or degenerate as in the above papers, so that things may be a quite "complicated" matter of time. Taking inspiration from decades of research on expected utility (outcome) risk preferences, this paper aims for a comprehensive account of time risk preferences and their implications within DEU.[4]

The next section sets the stage for studying risk preferences when things are only a matter of time, and shows that delay risk preferences are determined by the expectation of the discount factor. In section 3, I define the discount premium and characterize time risk preferences in the DEU

[4] There are many other important topics in the literature on time preferences that are complimentary and very different to this paper. These topics include the timing of the resolution of uncertainty (Kreps and Porteus, 1978), preferences towards intertemporal substitution and distinguishing them from risk preferences (Epstein and Zin, 1989), the empirical relationship between risk and time preferences (Andersen et al., 2008; Andreoni and Sprenger, 2012; Abdellaoui et al., 2013), time-inconsistency and decreasing impatience (Strotz, 1955; Laibson, 1997), and decision making under time pressure (Kocher et al., 2013), among others; see Epper and Fehr-Duda (2015) for a recent overview. There are also papers that study some form of "uncertain time", but which do not study *choices* over time risks. Dasgupta and Maskin (2005) show that the possibility of a premature arrival of a dated future outcome (i.e., a time risk) provides an evolutionary foundation of decreasing impatience. Yaari (1965) and Bommier (2006) study choices under uncertain lifetime and outcome utility of uncertain duration. The difference to the latter papers is that they consider utility maximization in the presence of *one, exogenous* risk that determines the number of the model's periods, while I – again – study choices over several (typically two) time risks themselves so that the time risk which the decision maker faces is endogenous.

model through risk apportionment. Afterward, I provide intuition for the time risk preferences implied by the commonly used discount functions. Section 4 studies several behavioral implications of time risk preferences, including saving and investment. Section 5 turns to the discounting of undesirable events that are a matter of time, derives the willingness to pay to avoid the time risk of catastrophe, and provides a generalized net present value formula for situations in which time risk cannot be ignored. The paper concludes with a discussion of the results and possible extensions to modeling behavior when things are – only or also – a matter of time.

2 The time-outcome risk duality

Consider a delayed outcome (τ, x) that describes a certain outcome $x > 0$ that is received at a certain time $\tau \geq 0$. Under DEU with smooth and strictly increasing utility function $u : [0, \infty) \longrightarrow [0, \infty)$ and smooth and strictly decreasing discount function $\rho : [0, \infty) \longrightarrow (0, 1]$ with $\rho(0) = 1$, (τ, x) is evaluated as[5]

$$U(\tau, x) := \rho(\tau)u(x). \tag{1}$$

A large literature generalizes this setting by assuming that the outcome x that is paid at time τ is risky, denoted by $\tilde{x}$, and evaluates the resulting *(delay-certain) outcome risk* $(\tau, \tilde{x})$ as

$$U(\tau, \tilde{x}) := \mathbb{E}[\rho(\tau)u(\tilde{x})]. \tag{2}$$

A DEU decision maker prefers the delay-certain outcome risk $(\tau, \tilde{x}_1)$ over the delay-certain outcome risk $(\tau, \tilde{x}_2)$ – note that it is dated to the same time τ – if and only if

$$\begin{aligned} U(\tau, \tilde{x}_1) \geq U(\tau, \tilde{x}_2) &\Longleftrightarrow \rho(\tau)\mathbb{E}[u(\tilde{x}_1)] \geq \rho(\tau)\mathbb{E}[u(\tilde{x}_2)]. \\ &\Longleftrightarrow \mathbb{E}[u(\tilde{x}_1)] \geq \mathbb{E}[u(\tilde{x}_2)]. \end{aligned} \tag{3}$$

We see that the delay τ together with the discount function $\rho(\cdot)$ drops out of the equation. It follows that preferences over risky outcomes dated to the same certain time are determined by the curvature of the utility function alone.

[5]To avoid technical difficulties, risky outcomes $\tilde{x}$ and risky delays $\tilde{\tau}$ are assumed to have their support contained in a finite subset of the domain of u and ρ, respectively. Moreover, we will only consider weak preference relations in this section and the next. Strict-preference analogs can be defined in the obvious way but require more complex modeling, with little extra in the way of economic insight. We will not study undesirable outcomes x until section 5. Appendix B presents a derivation of this section's result in a discrete-time version of the model with intermediate consumption. Appendix C considers a generalization where the desirable outcome has a duration period during which it grants utility. The simple model here is chosen to make the similarity between outcome risk preferences and delay risk preferences as transparent as possible.

Now we go back to the "delay-certain, certain outcome" (τ, x) and consider the mathematically symmetric generalization of making τ risky, thus denoting it by $\tilde{\tau}$, while letting the outcome x remain certain. The resulting object $(\tilde{\tau}, x)$ is an *(outcome-certain) time risk* or an *(outcome-certain) delay risk* for which the amount to be received is certain but the time it is received is not. In other words, the receipt of x is only a matter of time, and it is evaluated as:

$$U(\tilde{\tau}, x) := \mathbb{E}[\rho(\tilde{\tau})u(x)].$$

Next, let us focus on choices over two delay risks $(\tilde{\tau}_1, x)$ and $(\tilde{\tau}_2, x)$ with the *same* certain outcome $x > 0$. It follows that

$$\begin{aligned} U(\tilde{\tau}_1, x) \geq U(\tilde{\tau}_2, x) &\Longleftrightarrow u(x)\mathbb{E}[\rho(\tilde{\tau}_1)] \geq u(x)\mathbb{E}[\rho(\tilde{\tau}_2)] \\ &\Longleftrightarrow \mathbb{E}[\rho(\tilde{\tau}_1)] \geq \mathbb{E}[\rho(\tilde{\tau}_2)] \end{aligned} \tag{4}$$

and, therefore:

Proposition 1 (Time-outcome risk duality for desirable outcomes)**.** *Consider a DEU agent with preferences $\succeq$ over same-certain-delay outcome lotteries $(\tau, \tilde{x}_1)$ and $(\tau, \tilde{x}_2)$ resp. same-certain-outcome delay lotteries $(\tilde{\tau}_1, x)$ and $(\tilde{\tau}_2, x)$. For desirable outcomes $x > 0$ we have:*

$$(\tau, \tilde{x}_1) \succeq (\tau, \tilde{x}_2) \Longleftrightarrow \mathbb{E}[u(\tilde{x}_1)] \geq \mathbb{E}[u(\tilde{x}_2)] \tag{i}$$

$$(\tilde{\tau}_1, x) \succeq (\tilde{\tau}_2, x) \Longleftrightarrow \mathbb{E}[\rho(\tilde{\tau}_1)] \geq \mathbb{E}[\rho(\tilde{\tau}_2)]. \tag{ii}$$

The derivation of the time-outcome risk duality is a bit more subtle than the simple mathematics may suggest. Before I explain this, let me discuss the importance of the new equivalence (ii) in Proposition 1. First, sufficiency ("$\Longrightarrow$") means that choices over pure delay risks identify the discount function *independently from utility.* The utility function drops out of the equation. Second, necessity ("$\Longleftarrow$") tells us that an inequality over the *expected discount factors* of delay risks determines choices over delay risks. The intuition is that if the desirable outcome that is only a matter of time comes with a higher discount factor *in expectation* (so that it is discounted less), then it yields higher DEU. I put equivalence (ii) next to the well-known equivalence (i) in order to illustrate the similarity to the well-studied object of expected utility. The mathematical symmetry should prove useful in understanding preferences over delay risk or, equivalently, behavior when things are only a matter of time. While this will turn out to be true, the fact that utility functions are shaped quite differently than discount functions makes the study of delay preferences intricate.

Next, I will argue that – ex ante – the idea of understanding and characterizing preferences over *bivariate* delay risks by exploiting a similarity to *atemporal, univariate* expected utility preferences seems neither straightforward nor very promising. In particular, three important qualifications must be made in order to obtain the identification of discounting independently from utility through delay risks, as described in Proposition 1. First, the delayed outcomes must be non-risky ("$\tilde{x} \equiv x$").[6] Second, the delayed outcomes are time-independent, i.e., outcomes are not indexed to time ("$x \neq x_\tau$"). This assumption constitutes the major departure from much of the literature on time preferences which studies changes in impatience through trade-offs of the "more sooner" vs. "less later" type, as well as the moderating effect of outcome risk on this tradeoff. With any such tradeoff approach, one cannot arrive at a mathematical symmetry as described in Proposition 1, because utility will not drop out of the equations. This issue has been haunting the experimental literature on risk and time preferences (see Cheung 2016 for a discussion) until the recent development of the method proposed in Attema et al. (forthcoming). Third, choices are restricted to delay risks over the *same* outcome ("$x_1 = x_2 \equiv x$").

Therefore, the focus on choices over same-outcome, pure delay risks admits the identification of the discount function independently from utility. These risks precisely describe situations where "things are only a matter of time." For that reason, the identification of the discount function in Proposition 1 is not merely a mathematical artifact, but economically meaningful – pure delay risks even allow for an interpretation in terms of everyday language. Finally, and consistent with this interpretation, $\tilde{\tau}_1$ and $\tilde{\tau}_2$ in Proposition 1 are not restricted to binary and/ or degenerate risks. As such, we characterize delay risk preferences through choices where things may be a "complicated" matter of time.

With this in mind, let us clarify a possible confusion about the role of time in this paper. In particular, despite the mathematical similarity suggested by Proposition 1, this paper is not an expected utility paper that uses discount functions as utility functions. To see this, note that we conduct "the study of time as the duration that a person has to wait until she receives an outcome in intertemporal choice (discounting)" (Abdellaoui and Kemel, 2014)[p.1844]. Leclerc et al. (1995), Abdellaoui and Kemel (2014), Noussair and Stoop (2015), and Ebert and van de Kuilen (2015), on the other hand, study risk preferences over delay or waiting time itself. In particular, these authors conduct experiments where the outcome of subjects' decisions is waiting shorter or longer in a room without the possibility to do anything. In that case, time is an *outcome* and, therefore choices over risks in waiting time identify the *utility of "time"*. As Proposition 1 shows, choices over delay risks instead identify discounting (of a fixed amount of utility).[7] Therefore, this paper is

[6] In particular, this excludes the possibility of "something going wrong" (Halevy, 2008; Epper and Fehr-Duda, 2015), i.e., the outcome x not being paid after all. The assumption of outcome-certainty can be be relaxed, though, as long as outcome risk is independent of delay risk.

[7] As a concrete example, suppose that the outcome x describes a fixed relief in waiting time when ordering one's morning coffee (e.g., waiting four minutes less in line). A choice between waiting less in 2 days for sure and waiting less in either 1 or 3 days with equal probability is determined by the DM's discount function. The DM's utility function will determine whether, at any fixed time τ, she would prefer to wait 5 minutes for certain or to wait 2 or 8 minutes

fundamentally different from all papers that study curvature properties of utility functions, because these curvature properties have no meaning for choices over delay risks; only the properties that we define in the following do.

3 A behavioral characterization of time risk preferences when outcomes are desirable

This section characterizes risk-taking behavior when a fixed desirable outcome ($x > 0$) is only a matter of time. I define risk-averse, prudent, and temperate discounting in analogy to Eeckhoudt and Schlesinger (2006) through preferences over simple and intuitive "time risk apportionment" lotteries, and show that these behavioral definitions are equivalent to signing various derivatives of the discount function. I prove a general result on the time risk preferences implied by all commonly used discount functions: Time risk-taking can be understood as (i) "less delay being better" and (ii) a preference for combining "good" with "good".

3.1 Decreasing discount functions

Consider a pair of degenerate delay-outcome lotteries $(\tilde{\tau}_A^{(1)}, x)$ and $(\tilde{\tau}_B^{(1)}, x)$ that, at respective times $0 < \tilde{\tau}_A^{(1)} \equiv s < l \equiv \tilde{\tau}_B^{(1)}$ yield the same certain outcome $x > 0$ (with s and l denoting a sooner and a later date). The superscripts "$^{(1)}$" are used to indicate that these delay-outcome risks can be used to characterize first-order time "risk" preferences, i.e., the first derivative of the discount function. Later subsections will define lotteries with higher subscripts since they will be shown to characterize higher-order time risk preferences, i.e., higher-order derivatives of the discount function. Both first-order lotteries $(\tilde{\tau}_A^{(1)}, x)$ and $(\tilde{\tau}_B^{(1)}, x)$ are degenerate which is why they may be depicted as:

$$(\tilde{\tau}_A^{(1)}, x) \overset{1}{\longrightarrow} (s, x) \qquad\qquad (\tilde{\tau}_B^{(1)}, x) \overset{1}{\longrightarrow} (l, x)$$

Indeed, under the smoothness of the discount function assumed throughout, it follows from Proposition 1 that $\rho'(\tau) \leq 0$ for all $\tau \geq 0$ if and only if $(\tilde{\tau}_A^{(1)}, x) \succeq (\tilde{\tau}_B^{(1)}, x)$ for all s, l.

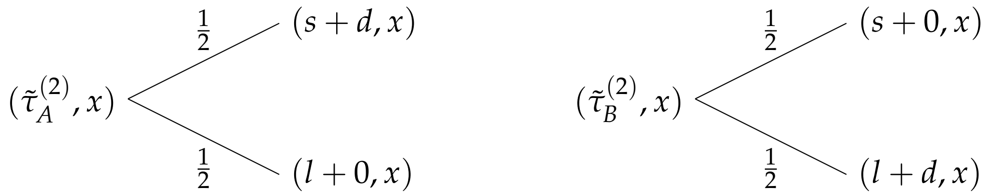
3.2 Risk-averse discounting

Onay and Öncüler (2007) and DeJarnette et al. (2015) have established the important result that the convexity of the discount function implies weak risk-seeking for outcome-certain time risks.

with equal probability. Recently, Abdellaoui et al. (2016) have conducted the first experiment that involves both types of "time". The authors elicit preferences towards the (non-risky) delay of (risky) outcome time.

In particular, using Jensen's inequality, the authors prove directly that a convex discount function implies that any outcome-certain time risk is preferred over its expected value. Using Proposition 1, this result also follows from Rothschild and Stiglitz (1970)[Theorem 2] for general mean-preserving spreads.[8]

Next, I show that a convex discount function is implied by a preference for combining "good" with "good". To see what this means, consider the following pair of lotteries



where $d > 0$ is some positive delay. Since less delay is better, as before, l is "bad" compared to the "good" s. Likewise, d is "bad" compared to the "good" non-delay of 0, which is depicted in the down-state of lottery $(\tilde{\tau}_A^{(2)}, x)$ precisely to make this connection salient. In this sense, $(\tilde{\tau}_A^{(2)}, x)$ is consistent with a preference for combining "good" with "bad" in each of its two states of nature while $(\tilde{\tau}_B^{(2)}, x)$ is consistent with a preference for combining "good" with "good" (in its up-state) as well as "bad" with "bad" (in its down-state). For sake of brevity, I will omit the "as well as 'bad' with 'bad' " part when unnecessary.

According to Proposition 1, the decision maker (DM) prefers to combine "good" with "good" in the above situation if and only if

$$\frac{1}{2}\rho(s+d) + \frac{1}{2}\rho(l) \leq \frac{1}{2}\rho(s) + \frac{1}{2}\rho(l+d)$$
$$\iff \rho(s+d) - \rho(s) \leq \rho(l+d) - \rho(l).$$

Dividing by d and letting d approach zero implies $\rho'(s) \leq \rho'(l)$ so that for l close to s, we obtain $\rho''(s) \geq 0$. It thus follows that a preference for combining "good" with "good" in the above setting for all s, l, d is equivalent to a convex discount function. I thus define:

Definition 1 (Risk-averse discounting)**.** *A DEU DM exhibits risk-averse (risk-seeking) discounting if and only if* $(\tilde{\tau}_A^{(2)}, x) \succeq (\preceq) (\tilde{\tau}_B^{(2)}, x)$ *for all* s, l, d.

By Proposition 1, convex discounting means that desirable outcomes are discounted *less* when they occur at an uncertain time as opposed to when they occur at the certain time with the same

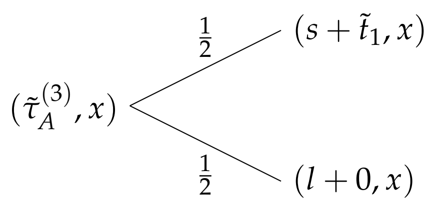
[8] Rothschild and Stiglitz (1970)[footnote 3] noted that: "It might be argued that we should limit our discussion to increasing concave functions. Imposing this restriction would gain nothing and would destroy the symmetry of the results." We see now that Rothschild and Stiglitz's general approach has also important *economic* consequences, since we can apply it to decreasing and convex discount functions.

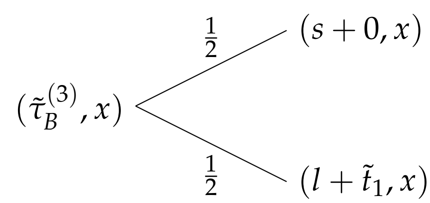
expected value. The risk apportionment definition of risk-seeking discounting given above, which was shown to be equivalent, says that an outcome is discounted relatively more upon increasing its delay when current delay is short as opposed to long.

Note that a preference for combining "good" with "bad" implies a concave discount function, which is incompatible with the discount function being positive (if it is indeed defined on the whole positive real line), as shown by DeJarnette et al. (2015). While the analysis presented here goes through irrespectively of this important observation, it encourages us to assume that risk-seeking discounting as predicted by the preference for combining "good" with "good" might be the economically more important concept.

3.3 Prudent discounting

Let $\tilde{t}_1$ and $\tilde{t}_2$ denote independent zero-mean delay risks and throughout let $s, l, \tilde{t}_1$ and $\tilde{t}_2$ be such that the support of final delay remains positive. Consider the preference over the following two lotteries that are obtained from the previous pair by replacing the fixed delay d with one of the zero-mean risks, say $\tilde{t}_1$:





A preference for $(\tilde{\tau}_A^{(3)}, x)$ over $(\tilde{\tau}_B^{(3)}, x)$ implies that the decision maker prefers to apportion an unavoidable zero-mean delay risk in the state where delay is "good". Consider a DM who prefers to combine "good" with "good". As we have seen in the previous section, her discount function must be convex so that she is delay risk-seeking. Therefore, an additional zero-mean delay risk is "good" relative to zero additional delay. Accordingly, she prefers $(\tilde{\tau}_A^{(3)}, x)$, which combines the "good" short delay s with the "good" zero-mean delay risk in the up state, over $(\tilde{\tau}_B^{(3)}, x)$. It is insightful to compare this behavior with a DM who prefers to combine "good" with "bad" so that – as shown in the previous subsection – his discount function is concave. For this delay risk-averse DM, a zero-mean delay risk is "bad." A preference for combining "good" with "bad" thus *also* implies a preference for $(\tilde{\tau}_A^{(3)}, x)$ over $(\tilde{\tau}_B^{(3)}, x)$. The intuition is that the risk-averse discounter can better cope with the "bad" zero-mean delay risk in the state where the fixed delay is shorter.

I now establish an equivalence between the preference of $(\tilde{\tau}_A^{(3)}, x)$ over $(\tilde{\tau}_B^{(3)}, x)$ and a curvature of the discount function. To this means, I define the *discount premium* D in analogy to the utility premium of Friedman and Savage (1948) as the change in the discount factor at delay τ that results from taking on a zero-mean time risk $\tilde{t}_1$:

$$D(\tau) := \mathbb{E}[\rho(\tau + \tilde{t}_1)] - \rho(\tau).$$

From our previous discussion, the discount premium is positive if and only if the discount function is convex. From Proposition 1, the preference for $(\tilde{\tau}_A^{(3)}, x)$ over $(\tilde{\tau}_B^{(3)}, x)$ is equivalent to

$$\begin{aligned} &\frac{1}{2}\mathbb{E}[\rho(s + \tilde{t}_1)] + \frac{1}{2}\rho(l) \geq \frac{1}{2}\rho(s) + \frac{1}{2}\mathbb{E}[\rho(l + \tilde{t}_1)] \\ &\quad \Longleftrightarrow D(s) \geq D(l), \end{aligned}$$

which is true for all $s < l$ if and only if the discount premium is decreasing: $D'(s) \leq 0$. We have

$$D'(s) \leq 0 \Longleftrightarrow \mathbb{E}[\rho'(s + \tilde{t}_1)] - \rho'(s) \leq 0,$$

which holds by Jensen's inequality if and only if ρ' is concave at τ, i.e., $\rho(\tau)''' \leq 0$. As in the utility paradigm (Crainich et al., 2013), we define prudent discounting as the time-risk lottery preference that follows from both the preference for combining "good" with "good" and the preference for combining "good" with "bad".[9]

Definition 2 (Prudent discounting)**.** *A DEU DM exhibits prudent (imprudent) discounting if and only if* $(\tilde{\tau}_A^{(3)}, x) \succeq (\preceq) (\tilde{\tau}_B^{(3)}, x)$ *for all* $s, l, \tilde{t}_1$.

Prudent discounting means that an outcome is discounted less upon increasing delay risk when current delay is low as compared to when it is high.

3.4 Temperate discounting

Fourth-order discounting preferences can be characterized by considering the following lottery pairs (note that now there is only one fixed delay, s):

$(\tilde{\tau}_A^{(4)}, x)$: with probability $\frac{1}{2}$ → $(s + \tilde{t}_1 + 0, x)$; with probability $\frac{1}{2}$ → $(s + 0 + \tilde{t}_2, x)$

$(\tilde{\tau}_B^{(4)}, x)$: with probability $\frac{1}{2}$ → $(s + \tilde{t}_1 + \tilde{t}_2, x)$; with probability $\frac{1}{2}$ → $(s + 0 + 0, x)$

[9] The deviation of the utility definition of prudence ($u''' \geq 0$) follows from the fact that discount functions are decreasing while the utility functions usually considered are increasing. For some arbitrary, smooth function f, $\text{sign}(f') \cdot f''' \geq 0$ gives thus the correct condition for prudence.

If the DM maker prefers to combine "good" with "good" ("good" with "bad"), then by subsection 3.2 the zero-mean risks are "good" ("bad") relative to the zero-delays. Accordingly, the former prefers $(\tilde{\tau}_B^{(4)}, x)$ while the latter prefer $(\tilde{\tau}_A^{(4)}, x)$. If preferences are in line with "good" with "good", by Proposition 1:

$$
\begin{aligned}
\frac{1}{2}\rho\mathbb{E}[(s+\tilde{t}_1)] + \frac{1}{2}\mathbb{E}[\rho(s+\tilde{t}_2)] &\leq \frac{1}{2}\mathbb{E}[\rho(s+\tilde{t}_1+\tilde{t}_2)] + \frac{1}{2}\rho(s) \\
\Longleftrightarrow \mathbb{E}[\rho(s+\tilde{t}_1)] - \rho(s) &\leq \mathbb{E}[\rho(s+\tilde{t}_1+\tilde{t}_2)] - \mathbb{E}[\rho(s+\tilde{t}_2)] \\
\Longleftrightarrow D(s) &\leq \mathbb{E}[D(s+\tilde{t}_1)].
\end{aligned}
$$

By Jensen's inequality, the last inequality is equivalent to a convex discount premium, i.e., $D''(s) \geq 0$. We have

$$D''(s) \geq 0 \Longleftrightarrow \mathbb{E}[\rho''(s+\tilde{t}_1)] - \rho''(s) \geq 0,$$

and invoking Jensen's inequality once more yields that ρ'' itself must be convex, i.e., $\rho'''' \geq 0$.

Definition 3 (Temperate discounting)**.** *A DEU DM exhibits temperate (intemperate) discounting if and only if* $(\tilde{\tau}_A^{(4)}, x) \succeq (\preceq)(\tilde{\tau}_B^{(4)}, x)$ *for all* $s, \tilde{t}_1, \tilde{t}_2$.

Temperate discounting is the trait that follows from a preference for combining "good" with "bad", which – as we showed – induces time risk aversion so that zero-mean risks are bad. In that case, temperance means that "bad" time risks are *mutually aggravating* (Kimball, 1993) so that the DM prefers to disaggregate them across states of nature. Intemperate discounting follows from a preference for combining "good" with "good" (which was shown to lead to time risk-seeking) and means that the "good" zero-mean risks are *mutually enticing.* Temperate discounting implies that adding delay to states where delay is risky already leads to a higher discount factor as compared to when adding it to states where it is not.

3.5 Summary and extension to orders higher than four

We have seen that that the signs of the first four derivatives of the discount function in the DEU model can be characterized by preferences over simple pure delay lottery pairs. These lotteries offer simple behavioral interpretations in terms of apportioning "good" or "bad" time risks as well as "bad" delays across two equally likely states of nature. Preferences for consistently combining "good" with "good" resp. "good" with "bad" imply identical signs for the derivatives of odd order, but different signs for the even orders. Having assumed that discount functions are decreasing ($\rho' < 0$), we showed that, when things are only a matter of time, a preference for combining "good" with "good" implies that DMs are risk-seeking ($\rho'' \geq 0$), prudent ($\rho''' \leq 0$), and intemperate

($\rho'''' \geq 0$). A preference for combining "good" with "bad" implies that DMs are risk-averse ($\rho'' \leq 0$), prudent ($\rho''' \leq 0$), and temperate ($\rho'''' \leq 0$) with respect to delay risk. Following the lottery nesting procedure outlined in Eeckhoudt and Schlesinger (2006), these results can be generalized to arbitrary odd and even orders. I leave the details of this for future research.[10]

3.6 The time risk preferences of "all commonly used discount functions"

It has long been noted that "[a]ll commonly used utility functions" Brockett and Golden (1987)[title] are increasing with derivatives that alternate in sign, a property later on called *mixed risk aversion* by Caballé and Pomansky (1996). Eeckhoudt et al. (2009) showed that this property is consistent with a preference for "more is better" and a "preference for combining good with bad". Let us call a discount function *anti-mixed (risk-averse)*[11] if it is *de*creasing with derivatives that alternate in sign. It is easily verified that all of the commonly used discount functions are indeed anti-mixed. In particular, I verified this for the geometric and exponential (Samuelson, 1937), Herrnstein's (1961) $\rho(t) = t^{-1}$, actually hyperbolic (Harvey 1986, 1995; Mazur 1987), generalized hyperbolic (Loewenstein and Prelec, 1992), and quasi-hyperbolic (Phelps and Pollak, 1968; Laibson, 1997), for all parameter values.[12] Ebert et al. (2016) show that all group discount functions (any weighted average of the discount functions mentioned above) are also anti-mixed.

Proposition 2 (The commonly assumed delay risk preferences towards desirable outcomes)**.** *When the outcome that is just a matter of time is desirable, the delay risk preferences of anti-mixed discount functions are consistent with "less delay is better" and a preference for combining "good" with "good".*

[10]Lajeri-Chaherli (2004) defines *edginess* as the fifth derivative of the utility function being positive. During his talk at the *Conference in Honor of Louis Eeckhoudt* in 2011, Miles Kimball suggested the term *bentness* to refer to a negative sixth derivative of the utility function. Harris Schlesinger, then, noted that earlier that day "someone at breakfast" already had suggested the term "kimballesque". As far as I can see, the scientific community has not settled on a terminology so far.

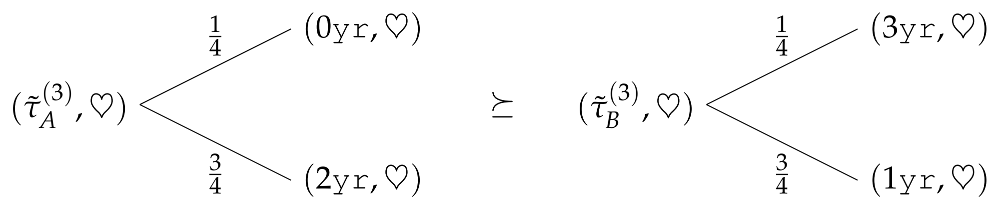
[11]Ebert (2013)[p. 1537] noted that some risk-loving power utility functions have this property and referred to them as "the opposite of mixed risk-averse". The term "mixed risk-seeking" (Crainich et al., 2013) is already taken and refers to a utility function with all derivatives being strictly positive.

[12]For a comprehensive overview of parametric forms of discount functions, see Bleichrodt et al. (2009). The two new discount functions proposed in that paper are also anti-mixed if one assumes decreasing impatience.

4 Applications

4.1 Prudent discounting: Downside delay risk aversion and delay skewness aversion

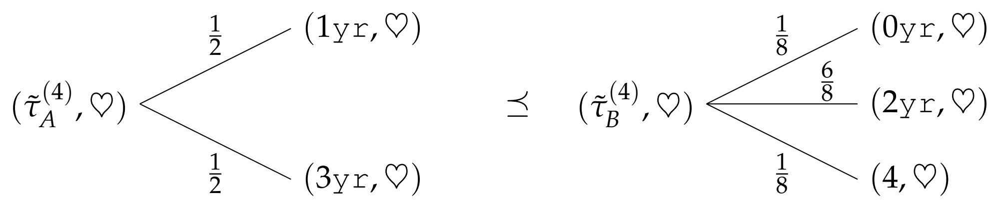
In this subsection and the next, I illustrate that the higher-order derivatives of the discount function determine the DM's preferences towards skewed and leptokurtic delay risks. Risk aversion (determined by the sign of ρ'') plays no role in this regard, but prudence and temperance do. For sake of concreteness, consider the following stylized example where $\texttt{yr}$ denotes the unit of a year and $x = \heartsuit$ refers to the positive outcome of a pregnancy test that a wannabe mother or father has long been waiting for:



The lotteries are chosen to be the analogues to the (atemporal) outcome risks in Mao (1970). The expected delay in both choices is identical to $1.5\texttt{yr}$ and both risks also feature the same variance. The skewness of $(\tilde{\tau}_A^{(3)}, \heartsuit)$, however, is negative while the skewness of $(\tilde{\tau}_B^{(3)}, \heartsuit)$ is positive so that the distribution of $(\tilde{\tau}_A^{(3)}, \heartsuit)$ is left-skewed while the distribution of $(\tilde{\tau}_B^{(3)}, \heartsuit)$ is right-skewed. Taking into account that less delay is good, $(\tilde{\tau}_B^{(3)}, \heartsuit)$ constitutes a *downside risk increase* of $(\tilde{\tau}_A^{(3)}, \heartsuit)$ as defined by Menezes et al. (1980). $(\tilde{\tau}_A^{(3)}, \heartsuit)$ features a small probability of no delay while $(\tilde{\tau}_B^{(3)}, \heartsuit)$ features a small probability of a very long delay. Formally, a downside risk increase comprises a mean-preserving delay spread where delay is bad (which is on the right tail of the distribution when less is good) together with a mean-preserving contraction in the region of good delays (on the left tail in this case) so that mean and variance remain unchanged. From Menezes et al. (1980) it follows that the DM is *downside risk averse* if and only if the discount function satisfies $\rho''' \leq 0$, i.e., if and only if the discount function is prudent as defined above. For a direct proof simply note that, by reduction of compound lotteries, the lottery pair above is a prudence lottery pair with $s = 1\texttt{yr}$, $l = 2\texttt{yr}$, and zero-mean risk $\tilde{t} = [1\texttt{yr}; -1\texttt{yr}]$. Prudence means that the mean-preserving spread $\tilde{t}$ of $0\texttt{yr}$ is placed where delay is better.

4.2 Intemperate discounting: Outer delay risk-seeking and kurtosis-seeking

Following the lines of Menezes and Wang (2005), one can show that temperance lotteries are *outer risk increases (contractions)* of one another, meaning that they have equal mean, variance, and skewness, but differ in their kurtosis. Just as a convex discount function implies risk-seeking, intemperate discounting implies outer-risk seeking. As an example, an intemperate discounter will exhibit the following preference:



4.3 Investment under delay risk

Consider investing in a risky start-up project that pays off once when exiting the company (say, through a leveraged buyout or an initial public offering). The exit (and payout) time $\tilde{\tau}$ is given by an uncertain future period with finite support contained in $\mathbb{N}_{>0}$. The current risk-free interest rate is zero. The investor has a baseline consumption of c today (at $t = 0$) as well as in each future period.[13] When investing a fraction $\alpha \in [0, c]$ of current wealth c in the project, utility today is reduced to $u(c - \alpha)$. Utility in the yet unknown exit period $\tilde{\tau}$ is increased by $u(c + \alpha\tilde{x})$, where $\tilde{x}$ is non-negative and independent from $\tilde{\tau}$. The agent's objective is to maximize

$$V(\alpha) = u(c - \alpha) + \sum_t \mathbb{P}[\tilde{\tau} = t] \left(\sum_{\tau \neq t} \rho(\tau)u(c) + \rho(t)u(c + \alpha\tilde{x}) \right),$$

which simplifies to (see also equation 20 in appendix B):

$$V(\alpha) = u(c - \alpha) + u(c) \sum_\tau \rho(\tau) + \mathbb{E}[\rho(\tilde{\tau})]\mathbb{E}[u(c + \alpha\tilde{x}) - u(c)].$$

The first-order condition is given by

$$V'(\alpha) = -u'(c - \alpha) + \mathbb{E}[\rho(\tilde{\tau})]\mathbb{E}[\tilde{x}u'(c + \alpha\tilde{x})] = 0,$$

and assuming concave u the second-order condition is satisfied. Let us compare the investment decision under different payout time distributions $\tilde{\tau}_i$ and denote the corresponding objective functions by V_i and the optimal investment level by α_i^* ($i = A, B$). For arbitrary α, since $\tilde{x}$ has positive support and since marginal utility is positive,

$$\mathbb{E}[\rho(\tilde{\tau}_A)] \leq \mathbb{E}[\rho(\tilde{\tau}_B)] \iff V_A'(\alpha) \leq V_B'(\alpha) \implies \alpha_A^* \leq \alpha_B^*,$$

[13]The assumption of constant baseline assumption is common in the literature and serves to isolate the effects of time risk in this case. The real options and optimal stopping literature (e.g., Dixit and Pindyck (1994) also makes this assumption, or operates in the setting laid out in the section 2.

where the last conclusion follows from the fact that V_i' (i=A,B) is decreasing so that $V_A'(\alpha)$ intersects with zero at a smaller value of α. Therefore, the first-order condition of the maximization problem is fulfilled at a smaller value of α. We thus see that the discount function alone determines the investment decision in this case. If ρ is anti-mixed, the DM will prefer the project with an exit time that is on average earlier than (a mean-preserving spread of, a downside risk decrease of, an outer risk increase of) a comparison project because his discount function is decreasing (risk-seeking, prudent, intemperate).

Note that an important assumption here was that the payout $\tilde{x}$ is time-independent. For example, if longer start-up investments pay more on average, then the discount and utility function will jointly determine the investment decision, even when two investments only differ in their exit times $\tilde{\tau}_1$ and $\tilde{\tau}_2$. Moreover, the example illustrates that the discount function may still determine choices independently from utility when the outcome whose delay is studied is risky. Here, this was the case because outcome risk was assumed to be independent from delay risk. Indeed, all results in this paper are easily generalized to risky outcomes that are independent from its risky delay.

4.4 Comparative risk-seeking over time risk and the Arrow-Pratt measure of a discount function

Let us define the *delay risk premium* of a zero-mean risk $\tilde{\tau}$ at current delay t as $\pi_d(t)$ which is the solution to

$$\rho(t - \pi_d(t)) = \mathbb{E}[\rho(t + \tilde{\tau})]. \tag{5}$$

Focusing again on the commonly used (decreasing, risk-averse, prudent,...) discount functions in this section, the interpretation is as follows. Receiving the desirable outcome x at time $t - \pi_d(t)$ for sure is as good as waiting for an additional $\pi_d(t)$ time units to receive the outcome sooner or later, i.e., with the additional "good" delay risk $\tilde{\tau}$. Following the analysis of de Finetti (1952), Pratt (1964), and Arrow (1965), for small risks $\tilde{\tau}$ with variance σ_τ^2 we obtain the approximation

$$\pi_d(t) \approx \frac{1}{2}\sigma_\tau^2 A_\rho(t) \tag{6}$$

where $A_\rho(t) = -\frac{\rho''(t)}{\rho'(t)}$ denotes the Arrow-Pratt measure of the discount function ρ, i.e., the coefficient of absolute delay-risk seeking. For the commonly used discount functions $A_\rho(t) > 0$ so that also $\pi_d(t) > 0$. The Arrow-Pratt measure of a discount function thus describes the DM's *willingness to wait (WTW)* for the desirable risk $\tilde{\tau}$. I state the following result without proof, because with

Proposition 1 in mind it is a straightforward adaption from results in Pratt (1964); see also Gollier (2001)[p. 20-21]:

Proposition 3 (Comparative time-risk seeking towards desirable outcomes)**.** *DM 2 with discount function ρ_2 is more time risk-seeking than DM 1 with discount function ρ_1 in the sense of the following statements, all of which are equivalent:*

$$A_2(t) \geq A_1(t) \forall t, x. \tag{I}$$

$$\rho_2(t) \text{ is a convex transformation of } \rho_1(t). \tag{II}$$

$$\text{DM 2 accepts all time risks } \tilde{t} \text{ at } t \text{ that DM 1 accepts at } t\ \forall t, x. \tag{III}$$

$$\text{DM 2 is willing to wait more at } t \text{ for time risk } \tilde{\tau} \text{ than DM 1, i.e., } \pi_{d,2} \geq \pi_{d,1},\ \forall t, x. \tag{IV}$$

I close this section by analyzing the connection between discount prudence and *decreasing absolute time risk seeking (DATRS)*, i.e.,

$$\frac{\partial}{\partial t} A(t) \leq 0 \Longleftrightarrow \rho'''(t)\rho'(t) \geq (\rho''(t))^2. \tag{7}$$

Since $\rho' < 0$, it follows that prudent discounting ($\rho''' \leq 0$) is necessary for DATRS, similarly to expected utility where $u' > 0$ and outcome prudence ($u''' \geq 0$) are necessary for decreasing absolute risk aversion (DARA).

As shown in section 3.3, for the risk-seeking discounter, prudence means that a "good" zero-mean time risk is less beneficial in states of "bad" (i.e., large) delay. By property (IV) above, and in line with this intuition of discount prudence, a DATRS discount function implies that the willingness to wait to receive a time risk decreases with delay. DATRS for time risk is a stronger property than discount prudence. While a time risk itself becomes less desirable as time increases, the decrease in the discount factor from having to wait in order to receive the time risk also decreases as time increases. DATRS implies that the former "prudence effect" (large $\rho'''(t)\rho(t)$ in equation 7) dominates the latter diminishing sensitivity effect (captured by $(\rho''(t))^2$.).[14]

Many other properties than DARA such as "properness" (Pratt and Zeckhauser, 1987) or "standardness" (Kimball, 1993) have been shown to play important roles for risk-taking behavior towards outcome risk. Moreover, other intensity measures such as Kimball's (1990) measure of absolute prudence ($-\frac{u'''}{u''}$) or Crainich and Eeckhoudt's (2008) alternative ($\frac{u'''}{u'}$) are decisive for behavioral predictions under expected utility. Studying the meaning and importance of these prop-

[14]This is in analogy to the intuition of DARA in the usual expected utility setting, namely that the risk premium (the willingness to pay to avoid undesirable risk) decreases. Prudence implies that risk can be better coped with when wealth is higher and is thus necessary for the risk premium to decrease with wealth. DARA ensures that the prudence effect dominates the effect that the pain from paying for risk avoidance decreases with wealth, which, in itself, would lead to a higher risk premium.

erties for time risk preferences remains a topic for future research.

4.5 Precautionary patience: prudent behavior towards delay risk

Consider a DM who receives two fixed payments, each of size $x > 0$, one at time $s \equiv 0$ and one at time $l > 0$. Consider the opportunity to delay the first payment by $\hat{d}$ days in order to receive the second payment $\hat{d}$ days sooner. The DM's problem is to maximize her discounted expected utility

$$\max_{\hat{d} \in [0,l]} \rho(\hat{d}) u(x) + \rho(l - \hat{d}) u(x).$$

Since the discount function is convex, corner solutions prevail. The DM is indifferent between $\hat{d} = 0$ and $\hat{d} = l$, each of which grants a DEU utility of $u(x)(\rho(0) + \rho(l)$.[15] Intuitively, since the DM is time-risk-seeking, all that she wants is that both payments are as far apart from each other as possible (and she doesn't care whether she receives the "first" payment of x now and the "second" payment of x later at l, or vice versa).

Next, suppose that the time of when the future payment is made becomes uncertain so that l is replaced by $l + \tilde{t}$ where $\tilde{t}$ is some zero-mean risk. The DM's problem becomes

$$\max_{\hat{d} \in [0,l]} u(x) \left(\rho(\hat{d}) + \mathbb{E}[\rho(l - \hat{d} + \tilde{t})] \right).$$

The convexity of ρ again excludes interior solutions. For $\hat{d} = 0$, the DM's utility is given by

$$u(x) \left(\rho(0) + \mathbb{E}[\rho(l + \tilde{t})] \right)$$

while for $\hat{d} = l$ it is given by[16]

$$u(x) \left(\rho(l) + \mathbb{E}[\rho(\tilde{t})] \right).$$

The DM strictly prefers $\hat{d} = l$ over $\hat{d} = 0$ if

$$\frac{1}{2}\rho(l)u(x) + \frac{1}{2}\mathbb{E}[\rho(\tilde{t})]u(x) > \frac{1}{2}\rho(0)u(x) + \frac{1}{2}\mathbb{E}[\rho(l + \tilde{t})]u(x),$$

which describes a (strict) prudence lottery preference $(\tilde{\tau}_A^{(3)}, x)$ over $(\tilde{\tau}_B^{(3)}, x)$; cf. section 3.3. Therefore, the DM is willing to delay a delay-certain payment in order to receive a time-uncertain

[15] I assume that for $\hat{d} = \frac{l}{2}$ the DM either frames the payments narrowly (i.e., receives utility $u(x) + u(x)$ rather than $u(2x)$) or that u is concave so that $u(2x) \leq u(x) + u(x)$.

[16] For simplicity I ignore the issue of negative time, which is easily resolved by replacing current time $s = 0$ with some $s > 0$ and restricting the support of $\tilde{t}$ to $[-s, \infty)$.

payment sooner if and only if she is a strictly prudent discounter ($\rho''' < 0$). Recall that utility prudence is necessary and sufficient for precautionary saving, i.e., the willingness to increase saving towards uncertain states of nature.[17] Therefore, both the discount-prudent and utility-prudent behavior is precautionary in the sense that it results in a better constitution of the DM (higher wealth resp. less delay) in the state where wealth resp. time is risky.

5 Undesirable outcomes and the discounting of catastrophes that are only a matter of time

5.1 Adapting the previous results to undesirable outcomes

Having seen the results for desirable outcomes, let us now study risk preferences towards an undesirable outcome that is only a matter of time.[18] To this means, we extend the support of utility u to negative outcomes and let $u(0) = 0$. Then, an outcome x is undesirable if $x < 0$. In that case, inequality (4) flips in the derivation of the time-outcome risk duality when dividing by the negative utility of the undesirable outcome. Let us define the *dual discount function* $\hat{\rho}(t) := 1 - \rho(t)$ for all $t \in [0, \infty)$. Then:

Proposition 4 (Time-outcome risk duality for undesirable outcomes)**.** *Consider a DEU agent with preferences $\succeq$ over same-certain-delay outcome lotteries $(\tau, \tilde{x}_1)$ and $(\tau, \tilde{x}_2)$ resp. same-certain-outcome delay lotteries $(\tilde{\tau}_1, x)$ and $(\tilde{\tau}_2, x)$. For undesirable outcomes $x < 0$ we have:*

$$(\tau, \tilde{x}_1) \succeq (\tau, \tilde{x}_2) \Longleftrightarrow \mathbb{E}[u(\tilde{x}_1)] \geq \mathbb{E}[u(\tilde{x}_2)] \quad \text{(i)}$$

$$(\tilde{\tau}_1, x) \succeq (\tilde{\tau}_2, x) \Longleftrightarrow \mathbb{E}[\hat{\rho}(\tilde{\tau}_1)] \geq \mathbb{E}[\hat{\rho}(\tilde{\tau}_2)] \quad \text{(ii)}$$

Therefore, delay risk preferences towards undesirable events are determined by maximizing the expectation of the dual discount function $\hat{\rho}$. Note that the discount function describes the *discount factor* that is multiplied with the utility of the undesirable outcome. The dual discount function, in contrast, is equal to the *discount* of outcome utility that comes from receiving the outcome later than at time zero. A DEU decision maker thus chooses the alternative with the largest discount factor (with the largest discount) if and only if the outcome that is only a matter of time is desirable

[17] Crainich et al. (2013) demonstrate explicitly that such precautionary behavior is independent of the assumption of risk aversion or risk-seeking, i.e., of whether risk is “good” or “bad”, and determined by prudence alone, just as in the model above.

[18] Propositions 4 and 5 below are straightforward adaptions of Propositions 1 and 2, respectively. Because the case of undesirable outcomes is equally important, and for convenience of reference, I restate these adapted results in a stand-alone fashion and call them propositions rather than corollaries.

(undesirable). In other words, if the outcome is desirable (undesirable), he chooses the alternative where the outcome is discounted the least (the most).

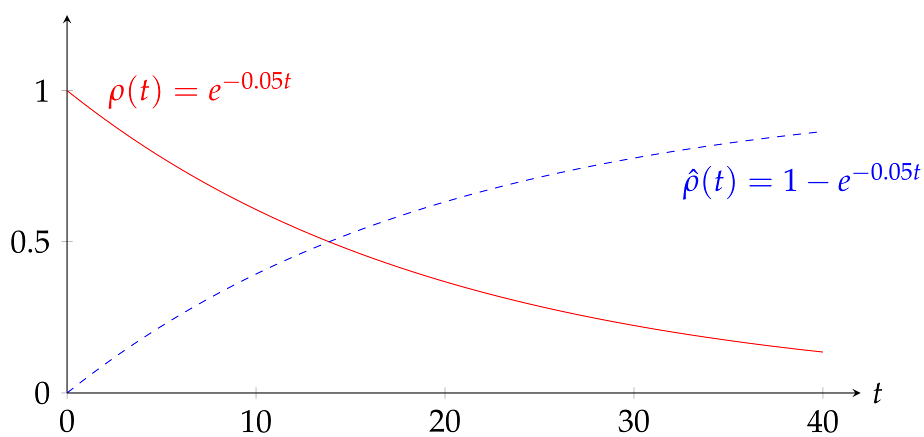
For anti-mixed discount functions ρ, the dual discount function $\hat{\rho}$ is mixed risk averse, i.e., increasing with derivatives that alternate in sign – just as it is the case for all commonly used utility functions. Therefore, because the shape of $\hat{\rho}$ is just like that of a "typical" u, delay risk preferences towards undesirable outcomes are similar to the well-studied risk-averse outcome risk preferences. In particular, from Proposition 4 and Eeckhoudt et al. (2009) it follows:

Proposition 5 (The commonly assumed delay risk preferences towards undesirable outcomes)**.** *Anti-mixed delay risk preferences towards undesirable outcomes are consistent with "more is better" and a preference for combining "good" with "bad".*

5.2 Exponential discounting of catastrophes that are only a matter of time

For exponential discounting, $\rho(t) = e^{-\delta t}$ for $\delta > 0$, the dual discount function is given by $\hat{\rho}(t) = 1 - e^{-\delta t}$; see Figure 1 for an illustration. For this important special case we thus make the observation that the dual discount function $\hat{\rho}$ is structurally identical to the well-known exponential (constant absolute risk-averse, CARA) utility function. From the equations in Proposition 4 it follows that risk preferences towards the delay of an undesirable outcome are like outcome risk preferences under constant absolute risk aversion. The level of absolute risk aversion is given by the rate of impatience: $A_{\hat{\rho}(t)} = \delta$.

Figure 1: The exponential (dual) discount function



Notes. Figure 1 plots an exponential discount function $\rho(t)$ with an impatience rate of $\delta = 0.05$ as well as the corresponding dual discount function $\hat{\rho} = 1 - \rho(t)$.

Therefore, now we know how a social planner with exponential discounting feels about the time risk associated with climate change. Because the case of exponential discounting is so important, let us examine carefully what constant absolute risk aversion towards time risk means. The indifference described by the equation (5), which defines the delay risk premium π_d, likewise holds for the dual discount function:

$$\hat{\rho}(t - \pi_d(t)) = \mathbb{E}[\hat{\rho}(t + \tilde{\tau})]. \tag{8}$$

Intuitively, the move from desirable to undesirable outcomes not only changes the preference towards certain delay (now, more delay is better) but also the preferences towards delay risk (now, less delay risk is better). Equation (8) thus says that facing an undesirable outcome by a time π_d earlier (which is bad) but at a known time (which is good) is as preferable as facing the undesirable outcome later (which is good) but at an uncertain time (which is bad). As in the case of desirable outcomes, therefore, π_d is positive for anti-mixed discount functions. It's interpretation, however, is that of a *willingness to accelerate* the occurrence of the undesirable outcome. For exponential discounting, the Arrow-Pratt approximation, equation (6), becomes

$$\pi_d(t) \approx \frac{1}{2}\sigma_\tau^2 A_{\hat{\rho}}(t) = \frac{1}{2}\sigma_\tau^2 A_\rho(t) = \frac{1}{2}\sigma_\tau^2 \delta. \tag{9}$$

Exponential discounting of a catastrophe that is only a matter of time thus implies that the willingness to accelerate the catastrophe (by an absolute number of days) is independent of its expected time of occurrence. As a catchy illustration, consider a cheating husband who knows that his wife finding out is only a matter of time. His willingness to accelerate her finding out in exchange for knowing when its going down is independent of whether he expects her to learn about his cheating in, say, one or two month's time.

5.3 The net present value (NPV) rule under time risk

The net present value (NPV) is the most widely used tool in investment analysis. The NPV rule says that an investment to prevent a future loss should be taken if and only if the cost of the investment is lower than its NPV. The NPV of avoiding a catastrophe further coincides with the willingness to pay (WTP) to avoid the catastrophe if the social planner is risk-neutral. In this section, we study a social planner's WTP to avoid an unavoidable catastrophe that is only a matter time. In particular, we study how this WTP changes under different time risks. For the special case where the social planner is risk-neutral, we will offer a generalized version of the seminal NPV formula that incorporates time risk.

Consider a social planer who faces a catastrophe that will occur at a non-degenerate, random time $\tilde{\tau}_2$ with finite support in $\mathbb{N}_{>0}$. She has a baseline consumption of c today (at $t = 0$) as well as in each future period $t = 1, 2, \ldots$. At time $\tilde{\tau}_2$, consumption is reduced by $0 < L < c$.[19] Suppose, first, that the social planner cannot prevent the catastrophe from happening, but she can invest today in a technology that will reduce the associated time risk $\tilde{\tau}_2$ to some other time risk $\tilde{\tau}_1$. In particular, $\tilde{\tau}_1 = \mathbb{E}[\tilde{\tau}_2]$ is the special case where time risk is removed entirely. Both time risks have finite support as well as (for simplicity) an integer expectation. Let

$$v(t) := \sum_{\tau \neq t} \rho(\tau) u(c) + \rho(t) u(c - L) \tag{10}$$

denote the discounted utility of future consumption (i.e., of consumption from period $t \geq 1$ onwards) given that the catastrophe hits at time t. Then, the planner's WTP for this technology, π, is defined implicitly via

$$u(c - \pi) + \sum_t \mathbb{P}[\tilde{\tau}_1 = t] v(t) = u(c) + \sum_t \mathbb{P}[\tilde{\tau}_2 = t] v(t). \tag{11}$$

Using equation (20) in appendix B, the above equation can be rearranged to become

$$\mathbb{E}[\rho(\tilde{\tau}_2)] - \mathbb{E}[\rho(\tilde{\tau}_1)] = \frac{u(c) - u(c - \pi)}{u(c) - u(c - L)}. \tag{12}$$

The right-hand side of this equality we denote by $\Delta u(\pi) = \frac{u(c) - u(c-\pi)}{u(c) - u(c-L)}$, which is a strictly increasing one-to-one mapping of π. Moreover, $\Delta u(\pi) > 0 \iff \pi > 0$ as long as u is increasing, and therefore

$$\mathbb{E}[\rho(\tilde{\tau}_2)] > \mathbb{E}[\rho(\tilde{\tau}_1)] \iff \pi > 0. \tag{13}$$

The inequality on the left-hand side is familiar by now, and equivalence (13) shows that it also determines the WTP to reduce the time risk of a catastrophe. As an example, when $\tilde{\tau}_2$ has more downside risk than $\tilde{\tau}_1$ (like when $\tilde{\tau}_2 = \tilde{\tau}_B^{(3)} + 1\texttt{wk}$ and $\tilde{\tau}_1 = \tilde{\tau}_A^{(3)} + 1\texttt{wk}$ as in section 4.1), then discount prudence ensures a positive WTP, $\pi > 0$. As another example, convex discounting is equivalent to a positive WTP for facing the catastrophe at a certain time over facing it at an uncertain time with equal expectation, which follows from equation (13) when $\tilde{\tau}_1 \equiv \mathbb{E}[\tilde{\tau}_2]$. Because we will refer to this case again later on, let us denote the premium of eliminating a time risk $\tilde{\tau}$ entirely

[19] The assumption of the damage L being certain is made so as to study the impact of time risk in isolation. Similarly to the investment application in section 4.3, the damage done can be uncertain as long as this uncertainty is independent from the delay risk $\tilde{\tau}$.

(i.e., the case $\tilde{\tau}_2 = \tilde{\tau}$ and $\tilde{\tau}_1 = \mathbb{E}[\tilde{\tau}]$) by $\pi_{\tilde{\tau}}$, in which case equation (12) becomes

$$\mathbb{E}[\rho(\tilde{\tau})] - \rho(\mathbb{E}[\tilde{\tau}]) = \Delta u(\pi_{\tilde{\tau}}). \tag{T}$$

Next, let us study the WTP to avoid not only the time risk, but the catastrophe entirely. Note that avoiding the loss L entirely makes the realization of the catastrophe's time risk irrelevant. In this sense, the WTP to avoid the loss L necessarily also eliminates the associated time risk, which is why we denote it by $\pi_{\tilde{\tau}L}$. It is defined via

$$u(c - \pi_{\tilde{\tau}L}) + \sum_t \rho(t) u(c) = u(c) + \sum_t \mathbb{P}[\tilde{\tau} = t] v(t), \tag{14}$$

where $v(t)$ is the discounted utility of future consumption given that the catastrophe hits at time t (recall equation 10). The above equation can be transformed into

$$\mathbb{E}[\rho(\tilde{\tau})] = \Delta u(\pi_{\tilde{\tau}L}). \tag{TL}$$

Before we interpret equation (TL), note that avoiding the catastrophe entirely in the absence of time risk, $\tilde{\tau} \equiv \mathbb{E}[\tilde{\tau}]$, yields a WTP (which we denote by π_L) given by

$$\rho(\mathbb{E}[\tilde{\tau}]) = \Delta u(\pi_L). \tag{L}$$

By combining equations (TL), (L), (T) one can observe that

$$\Delta u(\pi_{\tilde{\tau}L}) = \Delta u(\pi_L) + \Delta u(\pi_{\tilde{\tau}}), \tag{15}$$

i.e., the WTP to prevent a catastrophe that would hit at an uncertain time can be additively decomposed into a component attributable to the loss itself and a component attributable to the associated time risk. This result holds for all increasing utility functions, but it is most crisply illustrated for the case of a risk-neutral social planner. This case is also economically important, since then the WTP coincides with the net present value (NPV) of avoiding the catastrophe. In particular, note that under risk-neutrality we have $\Delta u(\pi) = \frac{\pi}{L}$. Then,

$$\pi_{\tilde{\tau}L} = \pi_L + \pi_{\tilde{\tau}}, \tag{NPV}$$

and the respective contributions are given by

$$\begin{aligned}\pi_{\tilde{\tau}L} &= L \cdot \mathbb{E}[\rho(\tilde{\tau})] \\ \pi_L &= L \cdot \rho(\mathbb{E}[\tilde{\tau}]) \\ \pi_{\tilde{\tau}} &= L \cdot (\mathbb{E}[\rho(\tilde{\tau})] - \rho(\mathbb{E}[\tilde{\tau}]))\,.\end{aligned}$$

π_L is the NPV as usually considered in the literature, i.e., the NPV of not incurring the loss of L in the (certain) period $\mathbb{E}[\tilde{\tau}]$. It is simply given by the loss itself, discounted by the time at which it will occur. In the presence of time risk and when using any of the commonly used (anti-mixed) discount functions, this NPV needs to be adjusted upward. The ignorance of time risk thus results in systematic underinvestment in measures against climate change. The magnitude of the bias is given by the WTP to remove the timing uncertainty without removing the loss itself, i.e., by $\pi_{\tilde{\tau}}$. As can be seen, $\pi_{\tilde{\tau}}$ is increasing in the convexity of the discount function as well as in the spread of the time risk.

Underestimating the NPV of climate change abatement due to the ignorance of time risk may result in the false allocation of the social planner's resources today. To the best of my knowledge, the environmental literature so far has ignored time risk altogether. The current paper explains how it is incorporated in cost-benefit analysis, simply by adding $\pi_{\tilde{\tau}}$ to the NPV.

6 Concluding remarks

This paper has studied risk preferences in the DEU model when things are only a matter of time. In other words, this paper has studied how outcomes that occur at uncertain times ought to be discounted. Choices in such situations are determined by the discount function alone. I defined risk-seeking discounting, prudent discounting, and intemperate discounting by preferences over simple delay lotteries, and showed that these preferences characterize the curvature of the discount function by signing its first four derivatives. All commonly used discount functions imply delay risk aversion, prudence, and temperance. Further, I characterized the Arrow-Pratt measure of a discount function through comparative time risk aversion and analyzed the behavioral implications of time risk preferences in various economic contexts.

It turns out that, when the outcome that is just a matter of time is undesirable, the delay risk preferences of commonly used discount functions are similar to the commonly assumed risk-averse outcome preferences. Then, well-known and important results from expected utility theory find a straightforward correspondence in the paradigm of time risk. I have given a new characterization of exponential discounting, namely constant absolute risk aversion towards the delay risk of a catastrophe. When a desirable outcome, on the other hand, is only a matter of time, then the translation of atemporal EU results to time risk is more difficult. After all, "the bible for future finance theory and practice" (Gollier, 2001)[quote by Paul A. Samuelson, back cover] is an insightful but difficult book to read, and translating some passages will require more work than others. This paper has translated some of the most popular parts, but many others remain to be done.

The current paper has pointed out the assumptions that are necessary for time risk preferences being determined by the discount function alone. These assumptions describe a situation where things are only matter of time. The fact that this phrase is commonly used in everyday language illustrates that these assumptions are natural and met in some real-world settings. At the same time, very clearly, there are also real-world applications where things are not only a matter of time. Most applications will involve both delay and outcome risk. As long as outcome and delay risk are independent, however, the results in this paper remain unaffected. Otherwise – if time and outcome risk correlate – neither the results in this paper nor the classical results on outcome risk preferences (as outlined in, e.g., Gollier 2001) determine behavior entirely. The recent literature on multivariate risk apportionment (e.g., Eeckhoudt et al. 2007) may prove useful in studying such cases. In any case, the results of the current paper will also play their role in situations with uncertainty about what will happen and when it well happen. Similarly, our knowledge of pure outcome risk preferences will continue to be important when taking into account time risk.

This paper has pointed out a similarity between the time and outcome domain as regards preferences towards risk. There are, however, a number of peculiarities to the time domain. Other behavioral traits such as time-inconsistency are also determined by the discount function alone. It thus seems interesting to focus on the time domain and study links between delay risk preferences and these traits. For example, stationary and time-inconsistent preferences imply exponential discounting (Halevy, 2015) and thus prudence (downside delay risk aversion).

A further suggestion for future research is the study of time risk in more general models than DEU, as done by Onay and Öncüler (2007) and DeJarnette et al. (2015). In appendix A, I show that the time-outcome risk duality can be generalized to rank-dependent utility (Quiggin, 1982). It then follows, from Wakker's (1994) seminal result, that *increasing marginal discounting* and *probabilistic time risk seeking* no longer coincide as they do in DEU. Also note that the definitions of prudent and temperate discounting in terms of lottery preferences are, in fact, theory-free. For outcome risk, Eeckhoudt et al. (2015) characterize the derivatives of the probability weighting function in Yaari's dual theory through simple lottery preferences. Baillon (forthcoming) applied the risk apportionment idea to ambiguity. Similar results may be possible for delay risk. Another, different generalization of the results in this paper is to allow for outcomes that come with a duration, i.e., that bring (dis-)utility not only once but for a pre-specified period of time. I discuss this generalization in appendix C. For exponential discounting I show that it is immaterial whether the outcome comes with a duration or not.

On the empirical side, it has been well recognized that the relative simplicity of the risk apportionment lotteries is amenable for experimentation; see Deck and Schlesinger (2014), Ebert and Wiesen (2014), Noussair et al. (2014), and Kocher et al. (2015) for recent contributions. In a companion paper, Ebert (2016), I test for risk-averse, prudent, and temperance discounting by making use of the fact that the lotteries defined in the current paper identify these properties independently from utility. In general, there seems to be an imbalance between experimental work on time risk preferences and outcome risk preferences. Further evidence on time risk preferences seems desirable in order to complement the vast existing evidence on time preferences that builds upon changes in impatience, i.e., the aversion towards *constant* delay. In conclusion, a number of interesting questions (theoretical and empirical, normative and descriptive) remain to be answered when things are a matter of time – and maybe also of something else.

A Time-outcome-risk duality with probability weighting

The time-outcome risk duality, Proposition 1, is readily extended to rank-dependent utility (Quiggin, 1982). To see this, focus on desirable outcomes and recall that the discounted rank-dependent expected utility (DRDU) of (τ, x) can be written as

$$U(\tau, x) = \int_{\mathbb{R}_+} w\left(\mathbb{P}(\rho(\tau)u(x) > y)\right) \mathrm{d}y \tag{16}$$

for some strictly increasing weighting function $w : [0, 1] \longrightarrow [0, 1]$ with $w(0) = 0$ and $w(1) = 1$. For a payoff-certain time risk $(\tilde{\tau}, x)$ we obtain

$$U(\tau, x) = \int_{\mathbb{R}_+} w\left(\mathbb{P}(\rho(\tilde{\tau}) > \frac{y}{u(x)})\right) \mathrm{d}y, \tag{17}$$

and by substituting $z := \frac{y}{u(x)}$ it follows that

$$U(\tau, x) = u(x) \int_{\mathbb{R}_+} w\left(\mathbb{P}(\rho(\tilde{\tau}) > z)\right) \mathrm{d}z =: u(x)RDU[\rho(\tilde{\tau})]. \tag{18}$$

From equation (18) it follows that the expected utility functional in Proposition 1 can be replaced with the rank-dependent one. We might now wish to apply Wakker's (1994) seminal result in order to separate *increasing marginal discounting* from *probabilistic time risk seeking.* Strong and weak risk aversion towards time risk in DRDU can be characterized through Chew et al. (1987) and Chateauneuf and Cohen (1994). For example, since discount functions are typically convex, risk-seeking in the strong sense of Rothschild and Stiglitz (1970) holds if and only if the weighting function is concave. Since commonly used weighting functions are not globally concave, this result is the basis for the fact that DRDU can explain the empirical evidence for time risk aversion. Further interesting insights may be obtained by extending equation (18) so as to incorporate an inherently risky future as suggested by Halevy (2008).

B Discrete time and intermediate consumption

Here, I show that the time-outcome risk duality can likewise be formulated in a discrete-time setting with intermediate consumption, similar to that of Onay and Öncüler (2007) or DeJarnette et al. (2015). Let us identify the possible time periods with the set of natural numbers. A consumption path is denoted by $\mathbf{c} = (c_1, \ldots, c_t, \ldots)$ and specifies consumption in each period with $c_t \geq 0$. Fix a base level of consumption c.[20] For each monetary prize $x \neq 0$, $x \geq -c$, and time $t \in \mathbb{N}$, let (t, x) now denote the consumption path that pays $c + x$ in period t and c in any other period. The pure delay risk $(\tilde{\tau}, x)$ yields the additional payment of x in one of finitely many periods t, with probability $\mathbb{P}[\tilde{\tau} = t]$, and is evaluated as

$$U(\tilde{\tau}, x) = \sum_t \mathbb{P}[\tilde{\tau} = t] \left(\sum_{\tau \neq t} \rho(\tau) u(c) + \rho(t) u(c + x) \right) \tag{19}$$

where $u : [0, \infty) \to [0, \infty)$ is a smooth and strictly increasing utility function and $\rho : \mathbb{N} \to (0, 1]$ is a strictly decreasing discount function. Note that the outcome x which is only a matter of time is desirable if and only if $x > 0$. Rewriting equation (19) yields

$$\begin{aligned} U(\tilde{\tau}, x) &= \sum_t \mathbb{P}[\tilde{\tau} = t] \left(\left(\sum_\tau \rho(\tau) - \rho(t) \right) u(c) + \rho(t) u(c + x) \right) \\ &= \sum_t \mathbb{P}[\tilde{\tau} = t] \left(\sum_\tau \rho(\tau) \right) u(c) + \sum_t \mathbb{P}[\tilde{\tau} = t] \left(-\rho(t) u(c) + \rho(t) u(c + x) \right) \\ &= 1 \cdot \sum_\tau \rho(\tau) u(c) + \mathbb{E}[\rho(\tilde{\tau})] \left(u(c + x) - u(c) \right), \end{aligned} \tag{20}$$

saying that the value of the time risk is given by the discounted utility of baseline consumption plus the change in utility from receiving the extra x, discounted at the uncertain time $\tilde{\tau}$. A choice

[20] The assumption of a constant baseline consumption is common in the time preferences literature. Here, it has the objective to isolate the impact of time risk preferences on decisions. Without this assumption, the impact of time risk preferences will be diluted by assumptions about changes in consumption (see also Onay and Öncüler 2007 or DeJarnette et al. 2015). Also the literature on search or real option exercise behavior fixes a baseline consumption or assumes the discounted utility model in the main text; see, e.g., the book of Dixit and Pindyck (1994).

between two same-outcome pure delay risks $(\tilde{\tau}_1, x)$ and $(\tilde{\tau}_2, x)$ is thus determined by

$$U(\tilde{\tau}_1, x) \geq U(\tilde{\tau}_2, x) \iff \mathbb{E}[\rho(\tilde{\tau}_1)]\,(u(c+x) - u(c)) > \mathbb{E}[\rho(\tilde{\tau}_2)]\,(u(c+x) - u(c))$$
$$\iff \begin{Bmatrix} \mathbb{E}[\rho(\tilde{\tau}_1)] \geq \mathbb{E}[\rho(\tilde{\tau}_2)], & \text{if } x > 0 \\ \mathbb{E}[\rho(\tilde{\tau}_1)] \leq \mathbb{E}[\rho(\tilde{\tau}_2)], & \text{if } x < 0 \end{Bmatrix},$$

where the last equivalence illustrates the case distinction that must be made when dealing with desirable and undesirable outcomes, respectively. We thus have seen that the utility of baseline consumption drops out of the equation just like the utility of baseline consumption plus the desirable outcome does. Therefore, the time-outcome risk duality still holds.

C Outcomes with a duration

Referring to the notation in the previous appendix B, suppose that the event that is only a matter time does not only change utility to $u(c+x)$ at the realized time $\tilde{\tau} = t$, but also in the $s \in \mathbb{N}$ subsequent periods $t+1, \ldots, t+s$. For example, suppose that it is only a matter of time until you will happily fall in love (again), which will make you happier for $s+1$ periods in total.[21] Focusing for simplicity on desirable outcomes $x > 0$ again, a derivation similar to the one in appendix B shows that

$$U(\tilde{\tau}_1, x) > U(\tilde{\tau}_2, x) \iff \sum_{t=0}^{s} \mathbb{E}[\rho(\tilde{\tau}_1 + t)] \geq \sum_{t=0}^{s} \mathbb{E}[\rho(\tilde{\tau}_2 + t)]. \tag{21}$$

We now show that the duration for which an outcome grants utility is irrelevant for preferences over time risks if and only if the discount function is exponential. To see this, recall that for exponential discounting $\rho(t+s) = \rho(t) \cdot \rho(s)$ for all $s, t \in \mathbb{N}$. Therefore,

$$U(\tilde{\tau}_1, x) > U(\tilde{\tau}_2, x) \iff \sum_{t=0}^{s} \mathbb{E}[\rho(\tilde{\tau}_1)\rho(t)] \geq \sum_{t=0}^{s} \mathbb{E}[\rho(\tilde{\tau}_2)\rho(t)]$$
$$\iff \left(\sum_{t=0}^{s} \rho(t)\right) \mathbb{E}[\rho(\tilde{\tau}_1)] \geq \left(\sum_{t=0}^{s} \rho(t)\right) \mathbb{E}[\rho(\tilde{\tau}_2)] \iff \mathbb{E}[\rho(\tilde{\tau}_1)] \geq \mathbb{E}[\rho(\tilde{\tau}_2)]$$

so that, under exponential discounting, all results in this paper remain unchanged if outcomes have a duration during which they affect utility. In particular, consider the "rational" assessment of the disutility from climate change within the DEU model and exponential discounting, which underlies the book of Gollier (2014). If we assume that a consequence of climate change does not only

[21] Using Wald's equality, it is possible to extend the following result to uncertain durations $\tilde{s}$ that are independent to $\tilde{\tau}$.

decrease utility in one period, but also in several subsequent periods, then it is still true that increasing uncertainty about when climate change will harm us is welfare-decreasing. If discounting is not exponential, this effect will still be present but interact with others, as described by the general equivalence (21). In any case, note that the introduction of outcome duration does not change the fact that preferences over time risks are determined by the discount function alone.

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