**The willingness to pay for health improvements under comorbidity ambiguity**[1]

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**Abstract**

Accumulated medical information is necessary to determine comorbidity risk between primary and its additional diseases. However, medical decisions often have to be made before obtaining conclusive evidences because of lack of information. This paper describes such situation by introducing ambiguity into comorbidity uncertainty. This paper examines conditions that the willingness to pay for health improvements increases by the introduction of ambiguity comorbidity compared with the corresponding risk case.

Keywords: Ambiguity, Comorbidity, Cost-benefit analysis, Smooth ambiguity model

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## 1. Introduction

"Comorbidities" means a situation that individuals develop other diseases (secondary diseases) in addition to a specific disease (primary diseases) which individuals complain, simultaneously. The threat of comorbidities is increasing in more recent times because of aging societies. It is the first time for us to care large number of the aged. Since comorbidities tend to be appeared with ages, we have to treat particular symptoms with potential secondary diseases whose cases are not accumulated. In other words, information is scared to determine how we estimate the possibilities of comorbidities from particular symptoms of the aged.

How do we describe situations that aged individuals face potential comorbid threats under the lack of information? According to Cerreia-Vioglio *et al.* (2013), ambiguity is defined as "Ambiguity refers to the case in which a decision maker does not have sufficient information to quantify through a single probability distribution the stochastic nature of the problem he is facing." From the above definition, ambiguity seems to be a reasonable way to describe a situation that patients face potential comorbid threats. Information to quantify possibilities of a specific disease is more available than multiple diseases and their relations, thus the former is "more uncertain" than the latter.[3] The situations can approximately describe that risk is applied to the former case and ambiguity is applied to the latter case.

From the classical notion by Knight (1921) and the famous experiment by Ellsberg (1961), it is recognize to importance of ambiguity in the literature. There are many ambiguity models which have axiomatic foundations and describe observed choices. In a recent paper, Berger et al. (2013) introduced ambiguity into medical decision making. However, while they considered one source of uncertainty and it is ambiguity, we considered multiple sources of uncertainty and the level of uncertainty is different which describes the coexistence of risk and ambiguity. Bleichrodt *et al.* (2003) considers the willingness to pay for health improvements under comorbidity risk. This paper extends their analysis to comorbidity ambiguity.

[3] Uncertainty is used an umbrella word of both risk and ambiguity.

The organization of this paper is as follows. Section 2 describes the model. Section 3 examines the effect of comorbidity ambiguity on the willingness to pay for health improvements. In section 4, we collects the additional results of the main result. In section 5, we introduce ambiguity into the probability of primary disease which is positively related to the level of comorbidity. Section 5 makes concluding remarks.

**2. The model**

Let us consider an individual who displays particular symptoms. The symptoms are assumed to indicate four possible health status: The individual is healthy ($H_0$), catches the primary disease ($H_1 = H_0 - M_1$) and the secondary disease ($H_2 = H_0 - M_2$) individually, and the comorbidity that means both diseases being suffered simultaneously ($H_{12} = H_0 - M_{12}$). The severities are denoted $M_1$, $M_2$ and $M_{12}$, respectively. The health status is assumed to quantify the following order: $H_0 \geq H_1, H_2 \geq H_{12}$. Utility levels are determined at the wealth level ($W$) and the health status ($H$) as the following bivariate utility function:

$$U(W, H).$$

The wealth level is supposed to be constant for all health status. We suppose the utility function satisfy the following properties which are standard in the literature:

- $U_1, U_2 > 0$,
- $U_{11}, U_{22} \leq 0$,
- $U_{12} \geq 0$,

Here, the subscripts denote partial derivatives with respect to the first argument ($W$) and the second argument ($H$), for example, $U_1 = \partial U // \partial W$, $U_{22} = \partial^2 U / \partial H^2$, $U_{12} = \partial^2 U / \partial W \partial H$ and so on. We impose the following condition on the health status:

$$H_2 - H_{12} \geq H_0 - H_1. \quad (1)$$

(1) captures the health status worse when comorbidity is occurred compared that either disease is caught alone, because (1) can be rewritten $M_{12} \geq M_1 + M_2$. In other words, comorbidity reinforces each other. This condition is an interesting case and seems to describe reality of comorbidity. From $U_2 > 0$ and $U_{22} \leq 0$, we have

$$U(H_2) - U(H_{12}) \geq U(H_0) - U(H_1). \quad (2)$$

The above interpretation can be applied to the utility unit. In the reminder, we permit $W$ in the utility function if no confusion worries.

While there is a tendency of the positive relation between the initial disease and the comorbid disease, the definitive conclusion has not been obtained. In a word, the relation is ambiguous in the sense that it cannot be determined by a unique probability distribution. Formally, the probability of each health status is given:

- $H_0$ with probability $1 - p_1 - p_2 - k^i p_1 p_2$
- $H_1$ with probability $p_1(1 - k^i p_2)$
- $H_2$ with probability $(1 - k^i p_1)p_2$
- $H_{12}$ with probability $k^i p_1 p_2$

$p_1$ is the probability of the primary disease. The individual catches the primary disease in health status $H_1$ and $H_{12}$, so that $p_1 = p_1(1 - k^i p_2) + k^i p_1 p_2$. $p_2$ is the probability of the secondary disease. $k^i \geq 0$ is called (first-order) comorbid risk which is a parameter to determine the relation between the primary and the secondary diseases, where this parameter depends on index $i$. Bleichrodt *et al.* (2003) considered the comorbidity risk in the sense that $k$ is uniquely determined. In this paper, the uncertainty of comorbidity is not risk, but ambiguous. This paper extends their analysis from risk to ambiguity. $k^i$ being more (less than) than unity means positive (negative) correlation between the primary and the secondary diseases. There are two potential comorbid risk which are given by $k^h > k^l$. The probability that $k^h$ $(k^l)$ is true, is $q$ $(1-q)$. Even though the analysis is limited to the indices being binary for the simplicity, the analysis is extended to the general index case as presented in Appendix.

The individual's preference is assumed to exhibit the smooth ambiguity model from Klibanoff *et al.* (2005). Given comorbidity risk $k$, the (first-order) expected utility is written:

$$E^k[U] = kp_1p_2U(W,H_{12}) + p_1(1-kp_2)U(W,H_1) + (1-kp_1)p_2U(W,H_2)$$
$$+(1-p_1-p_2+kp_1p_2)U(W,H_0)$$

We use the notation $E^h[U]$ and $E^l[U]$ for $k=k^h$ and $k=k^l$. Applying this notation, the welfare of the individual is measured by:

$$V = q\phi\big(E^h[U]\big) + (1-q)\phi\big(E^l[U]\big).$$

$\phi$ is strictly increasing and concave, $\phi' > 0$ and $\phi'' \leq 0$. Concavity of $\phi$ captures ambiguity aversion in the sense that the individual dislikes a mean-preserving spread of the probability distribution over the expected utilities $E^h[U]$ and $E^l[U]$.

**3. Main Result**

Given $\phi$, we yield the willingness to pay (WTP) for health improvement as the following:

$$\mathrm{WTP}^{\phi} = \frac{dW}{dp_1} = -\frac{V_{p_1}}{V_W}.$$

It is calculated that

$$-V_{p_1} = q\phi'\big(E^h[U]\big)N^k + (1-q)\phi'\big(E^l[U]\big)N^l, \quad (3)$$

where $N^i = k^ip_2\big(U(H_2)-U(H_{12})\big) + \big(1-k^ip_2\big)\big(U(H_0)-U(H_1)\big)$ for $i=h,l$. We recall that $W$ is omitted in $U(W,H)$. It is also calculated

$$V_W = q\phi'\left(E^h[U]\right)E^h[U_1] + (1-q)\phi'\left(E^l[U]\right)E^l[U_1]. \quad (4)$$

Combining (3) and (4), the WTP is rewritten:

$$\mathrm{WTP}^{\phi} = \frac{q\phi'\left(E^h[U]\right)N^k + (1-q)\phi'\left(E^l[U]\right)N^l}{q\phi'(E^h[U])E^h[U_1] + (1-q)\phi'(E^l[U])E^l[U_1]}. \quad (5)$$

To examine the effect of ambiguity on the WTP, we set the expected utility case as the benchmark. We note that linear $\phi$ is degenerated into expected utility. When the individual is an expected utility maximizer, the WTP is given:

$$\mathrm{WTP}^{O} = \frac{N^O}{E^O[U]} = \frac{qN^k + (1-q)N^l}{qE^h[U_1] + (1-q)E^l[U_1]}. \quad (6)$$

Here, $N^O$ and $E^O[U]$ are calculated at $k^O = qk^H + (1-q)k^l$. The second equality is due to the linearity of expectation operator.

To determine the effect, we prepare the following lemma. The proofs is collected in Appendix 1.

**Lemma 1.**

1. $E^h[U] \leq E^l[U]$. (7)
2. $N^h \geq N^l$. (8)
3. If $U_{122} \leq 0$,

   $E^h[U_1] \leq E^l[U_1]$. (9)

We give an interpretation for all the results in Lemma 1, where it is noted that the uncertainty in Lemma 1 is risk determined by potential $k$. Because $k$ increases the probability of the comorbidity and because the comorbidity reinforces the severity each other, it is intuitive that the expected utility is worse for higher $k$. The fist result confirms that this intuition is right.

$-N^k = \partial E^k[U]/\partial p_1 < 0$ represents the disutility when the probability of the primary disease increases. (8) can be rewritten $-N^h \leq -N^l$, so (8) means that the individual suffers more disutility for higher $k$. This is reasonable because the primary disease tends to cause the secondary disease more when $k$ is higher. It is difficult to obtain intuitive understandings for (9). However, it is worth mentioning an interpretation for a negative value of $U_{122}$ that guarantees (9). Eeckhoudt *et al.* (2007) call $U_{122} \leq 0$, cross imprudence in wealth. Cross imprudence in wealth can interpret a location preference for a zero-mean health risk that an individual dislikes because of $U_{22} < 0$. For an individual who is cross prudent in health, the individual prefers to accept a health risk in lower wealth. In other words, aversion to health risk increases in wealth. Even though this seems to be reasonable as a certain validity, there is no empirical evidence the sign of $U_{122}$. A negative value of $U_{122}$ is also appeared in Bleichrodt *et al.* (2003) and they discuss this condition.

We readily show that

$$\mathrm{WTP}^{\phi} = \frac{q\phi'\left(E^h[U]\right)N^h + (1-q)\phi'\left(E^l[U]\right)N^l}{q\phi'(E^h[U])E^h[U_1] + (1-q)\phi'(E^l[U])E^l[U_1]} \geq \frac{N^O}{E\left[U_O^h\right]} = \mathrm{WTP}^O$$

From the above lemma, we have the followings:

Because $E^h[U] \leq E^l[U]$ and $\phi' > 0$,

$$\phi'\left(E^h[U]\right) \geq 1 \geq \phi'\left(E^l[U]\right). \quad (10)$$

Here, we can normalize

$$\phi'\left(E^h[U]\right) + \phi'\left(E^l[U]\right) = E^h[U] + E^l[U]$$

because $\phi$ is unique up to an affine transformation.

From (10) and $N^h \geq N^l$, we have

$$q\phi'\left(E^h[U]\right)N^h + (1-q)\phi'\left(E^h[U]\right)N^l \geq qN^h + (1-q)N^l = N^O \quad (11)$$

From (10) and $E^h[U_1] \leq E^l[U_1]$, we have

$$q\phi'\left(E^h[U]\right)E^h[U_1] + (1-q)\phi'\left(E^l[U]\right)E^l[U_1] \leq qE^h[U_1] + (1-q)E^l[U_1] = E^O[U_1]. \quad (12)$$

Combining (11) and (12), we have

$$\mathrm{WTP}^{\phi} \geq \mathrm{WTP}^{O}$$

We summarize the above argument into the following result:

**Result 1:**

Let us consider an ambiguity averse individual and suppose that $U_{122} \leq 0$. Ambiguity increases the WTP, that is, $\mathrm{WTP}^{\phi} \geq \mathrm{WTP}^{O}$.

The result can be extended to the general setting, where there are many possible $k$s. The general result is found in Appendix 2.

The result indicates that we may underestimate the WTP if comorbidity ambiguity were not incorporated. Currently, it is in progress aging society that we have not experienced before. Because comorbidity is usually displayed with age, there exist potential comorbidity whose unique probability distributions cannot be available because of lack of information. We need to pay attention on the underestimate of the WTP, especially for ages, if ambiguity aversion is accepted as a normative criterion.

## 4. Additional Results

We obtain the following additional results which can be viewed as the corollaries of the main result.

**Result 2**

Let us consider an ambiguity loving individual and suppose that $U_{122} \leq 0$. Ambiguity decreases the WTP, that is, $\mathrm{WTP}^{\phi} \leq \mathrm{WTP}^{O}$.

Kocher *et al.* (2015) observed ambiguity seeking behavior in loss domains or low reduced probabilities. The situation may fit to describe medical decision makings when individuals face comorbidity ambiguity. If so, the implication of the result is reversed. That is, we overestimate the WTP under comorbidity ambiguity by ignoring ambiguity seeking behavior. It is noted that optimization is not appeared in the analysis, so we do not need to be worried about the second-order condition which both risk aversion and ambiguity aversion guarantee.

**Result 3**

Let us consider an ambiguity averse individual and suppose that $U_{122} \leq 0$. The WTP is increasing in $q$. Hence, the upper and lower bounds for WTP are given $\mathrm{WTP}^{l}$ and $\mathrm{WTP}^{h}$ which corresponds $k^{h}$ and $k^{l}$ are true certainly[4], that is,

$$\mathrm{WTP}^{h} \geq \mathrm{WTP}^{\phi} \geq \mathrm{WTP}^{l}.$$

Because

[4] $\mathrm{WTP}^{h} = \frac{N^{h}}{E^{h}[U_1]}$ and $\mathrm{WTP}^{l} = \frac{N^{l}}{E^{l}[U_1]}$.

$$\operatorname{sgn}\left\{\frac{\partial}{\partial q}\mathrm{WTP}^{\phi}\right\} = \operatorname{sgn}\{\phi'(E^h[U])(N^h - E^h[U_1]) - \phi'(E^l[U])(N^l - E^l[U_1])\},$$

the WTP is increasing in $q$ by Lemma 1. Because $\mathrm{WTP}^h$ and $\mathrm{WTP}^l$ correspond the case of $q = 1$ and $q = 0$ respectively, they give the upper and lower bounds for the WTP.

Let us consider that an agent whose preference is represented the maxmin expected utility. Because $\mathrm{E}[U^h] \leq \mathrm{E}[U^l]$ holds, the WTP under maxmin expected utility is $\mathrm{WTP}^h$, which is less than the WTP under the smooth ambiguity model.

**Result 4**

Let us consider two ambiguity averse individuals, $A$ and $B$, and suppose that $U \leq 0$. If individual $A$ is more ambiguity averse than $B$ and ceteris paribus, then $\mathrm{WTP}^A \geq \mathrm{WTP}^B$.

Let us consider that two ambiguity averse patients $A$ and $B$. The patient $A$ is more ambiguity averse than $B$ in the sense that there exists an increasing concave function $g$ such that $\phi_A = g \circ \phi_B$. To focus on changes in ambiguity aversion, the only difference between individual *A* and *B* is their ambiguity aversion.

We normalize $\phi_A'(E^h[U]) + \phi_A'(E^l[U]) = \phi_B'(E^h[U]) + \phi_B'(E^l[U])$ without loss of any generality.

From $\phi_A'(E[U]) = g'(\phi_B(E[U]))\phi_B'(E[U])$, we have

$$g'\left(\phi_B(E^h[U])\right) + g'\left(\phi_B(E^l[U])\right) = 1 \text{ and } g'\left(\phi_B(E^h[U])\right) \geq 1 \geq g'\left(\phi_B(E^l[U])\right).$$

From the above, we can apply the proof of the main result to examine the effect of changes in ambiguity aversion.

Because the maxmin expected utility corresponds the infinite ambiguity aversion in the smooth ambiguity model when the functional form is an exponential case. $\mathrm{WTP}^h \geq \mathrm{WTP}^\phi$ in Result 3 can also be shown by using Result 4.

## 5. Changes in Probability of Primary Disease

Up to now, we assume that an individual can collect enough information for uncertainty of the primary diseases and thus its uncertainty can describe risk. In other words, the probability of the primary disease is always $p_1$ regardless of the level of comorbidity. This assumption is relaxed in this section, the probability of the primary disease is also ambiguous in addition to comorbidity.[5] The probabilities of the primary disease is given as

$$p_1 = \begin{cases} p_1^h \text{ for } k = k^h \\ p_1^l \text{ for } k = k^l. \end{cases}$$

Here we assume that $qp_1^h + (1-q)p_1^l = p_1$ and $p_1^h > p_1^l$. The first assumption is imposed for the comparison. The second assumption reflects that an individual is more prone to get diseases when comorbidity risk is high, which seems to be a reasonable setting. We stand for expected utility $p_1^i, i = h, l$ as

$$E^i[U] = kp_1^i p_2 U(W, H_{12}) + p_1^i(1 - kp_2)U(W, H_1) + \left(1 - kp_1^i\right)p_2 U(W, H_2)$$
$$+\left(1 - p_1^i - p_2 + kp_1^i p_2\right)U(W, H_0).$$

We note that the superscript disappears for $N$ because $p_1$ is not contained in $N$,

$$N = kp_2\left(U(H_2) - U(H_{12})\right) + (1 - kp_2)\left(U(H_0) - U(H_1)\right)$$

In this setting, the WTP can be rewritten:

$$\mathrm{WTP}^{p_1} = \frac{q\phi'\left(E^h[U]\right) + (1-q)\phi'\left(E^l[U]\right)}{q\phi'(E^l[U])E^l[U_1] + (1-q)\phi'(E^l[U])E^l[U_1]}.$$

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[5] We can also introduce ambiguity into the probability of the secondary disease. However, we need to repeat a similar analysis to introduce it, so we maintain the assumption that its probability is uniquely determined.

We obtain the following lemma.

**Lemma 2.**

1. $E^h[U] \leq E^l[U]$. (13)
2. If $U_{122} \leq 0$,

   $E^h[U_1] \leq E^l[U_1]$. (15)

The proof is found in Appendix 3. By a similar argument in Section 3, we obtain

$$\mathrm{WTP}^{p_1} \geq \mathrm{WTP}^{O}.$$

Combining the main result in Section 3, we obtain the following:

$$\mathrm{WTP}^{\phi} = \mathrm{WTP}^{p_1,k} \geq \mathrm{WTP}^{p_1} \geq \mathrm{WTP}^{O}.$$

Here the superscripts represents the existence of ambiguity.

**Result 5:**

Let us consider an ambiguity averse individual and suppose that $U_{122} \leq 0$. There exists ambiguity on the probability of primary disease and comorbidity risk with $p_1^h \geq p_1^l$ and $k^h > k^l$. Ambiguity increases the WTP, that is, $\mathrm{WTP}^{\phi} \geq \mathrm{WTP}^{O}$.

When it is not available to obtain enough information on the uncertainty of the primary disease, it is a better description for comorbidity ambiguity that the probability of primary disease is dependent on comorbidity risk. In our binary setting of potential comorbidity risk, there are two cases, that is, the probability of primary disease with high comorbidity risk is higher (lower) than that of low comorbidity risk, $p_1^h \geq (\leq) p_1^l$ for $k^h > k^l$. We show that ambiguity aversion increases the WTP for the former case which seems to be more reasonable. In the latter case, the effect of ambiguity aversion on the WTP is indeterminate. For ambiguity seeking, the result is reversed, that is, ambiguity decreases the WTP.

## 4. Conclusion

In this paper, ambiguity is introduced to comorbidity, more precisely, the parameter to represent the relation between primary and secondary diseases. We show that such ambiguity increases WTP for health improvements by imposing some conditions on bivariate utility function of wealth and health.

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**Appendix 1:**

From (2), we obtain that

$$\operatorname{sgn}\left\{\frac{\partial E^k[U]}{\partial k}\right\} = \operatorname{sgn}\left\{U(H_0) - U(H_1) - \left(U(H_2) - U(H_{12})\right)\right\} \leq 0.$$

Because $E[U^k]$ is decreasing in $k$, $E^h[U] \leq E^l[U]$ for $k^h > k^l$.

From (2), we obtain that

$$\operatorname{sgn}\left\{\frac{\partial N^k}{\partial k}\right\} = \operatorname{sgn}\left\{U(H_2) - U(H_{12}) - \left(U(H_0) - U(H_1)\right)\right\} \geq 0.$$

Because $N^k$ is increasing in $k$, $N^h \geq N^l$ for $k^h > k^l$.

From (1) and $U_{122} \leq 0$, we obtain that

$$\operatorname{sgn}\left\{\frac{\partial E^k[U_1]}{\partial k}\right\} = \operatorname{sgn}\left\{U_1(W, H_0) - U_1(W, H_1) - \left(U_1(W, H_2) - U_1(W, H_{12})\right)\right\} \leq 0.$$

Because $E^k[U_1]$ is decreasing in $k$, $E^h[U_1] \leq E^l[U_1]$ for $k^h > k^l$.

**Appendix 2:**

The main result can be extended to the general setting.

There are $n$ possible $k^i \geq 0$ $(i = 1,2, \dots, n)$ whose probability is $q^i$. We assume that $k^1 < \cdots < k^n$. The WTPs for expected utility and smooth ambiguity model are given as:

$$\mathrm{WTP}^{\phi} = \frac{\sum_i q_i \phi'\left(E^i[U]\right) N^i}{\sum_i q_i \phi'(E^i[U]) E^i[U_1]}$$

$$\mathrm{WTP}^{O} = \frac{\sum_i q_i N^i}{\sum_i q_i E^i[U_1]}$$

Without loss of generality, we normalize

$$\sum_i q_i \phi'\left(E^i[U]\right) = 1 \left( = \sum_{\mathrm{i}} q_i \right).$$

This means that $q\phi'(E[U]) = \left(q_1 \phi'(E^1[U]), \dots, q_n \phi'(E^n[U])\right)$ can be viewed as probability vector.

From the binary results, we can obtain that

$$E^1[U] \geq \cdots \geq E^n[U].$$

Since $\phi'$ is decreasing,

$$\phi'(E^1[U]) \leq \cdots \leq \phi'(E^n[U]).$$

For every $i$ and $j$ with $i < j$,

$$\frac{q_j}{q_n} \leq \frac{q_j \phi'\left(E^j[U]\right)}{q_i \phi'(E^i[U])}.$$

This means that $q\phi'$ dominates $q$ in the sense of monotone likelihood ratio dominance which is a stronger than first-order stochastic dominance.

From the result of binary case, we obtain the followings:

- $N^k$ is decreasing in $k$, $N^1 \geq \cdots \geq N^n$
- $E^i[U_1]$ is increasing in $k$, $E^1[U_1] \leq \cdots \leq E^n[U_1]$ if $U_{122} \leq 0$

From this, we obtain

- $\sum_i q_i \phi'\left(E^i[U]\right) N^i \geq \sum_i q_i N^i$
- $\sum_i q_i \phi'\left(E^i[U]\right) E^i[U_1] \leq \sum_i q^i E^i[U_1]$

Combining these,

$$\mathrm{WTP}^{\phi} = \frac{\sum_i q_i \phi'\left(E^i[U]\right) N^i}{\sum_i q_i \phi'(E^i[U]) E^i[U_1]} \geq \frac{\sum_i q_i N^i}{\sum_i q^i E^i[U_1]} = \mathrm{WTP}^{O}$$

**Appendix 3:**

The expected utility given $p_1$ is written:

$$E_{p_1}[U] = kp_1p_2U(W,H_{12}) + p_1(1-kp_2)U(W,H_1) + p_2(1-kp_1)U(W,H_2) + (1-p_1-p_2-kp_1p_2)U(W,H_0).$$

From (1), we obtain that

$$\frac{\partial E_{p_1}[U]}{\partial p_1} = kp_2\big(U(W,H_{12}) - U(W,H_2)\big) + (1-kp_2)\big(U(W,H_1) - U(W,H_0)\big) < 0.$$

The marginal expected utility is wealth given $p_1$ is written:

$$E_{p_1}[U_1] = kp_1p_2U_1(W,H_{12}) + p_1(1-kp_2)U_1(W,H_1) + p_2(1-kp_1)U_1(W,H_2) + (1-p_1-p_2-kp_1p_2)U_1(W,H_0)$$

From (1), we obtain that

$$\frac{\partial E_{p_1}[U_1]}{\partial p_1} = kp_2\big(U_1(W,H_{12}) - U_1(W,H_2)\big) + (1-kp_2)\big(U_1(W,H_1) - U_1(W,H_0)\big) < 0$$

by $U_{12} > 0$ .