Dual Moments and Risk Attitudes*

Louis R. Eeckhoudt
IÉSEG School of Management
Catholic University of Lille
and CORE
Louis.Eeckhoudt@fucam.ac.be

Roger J. A. Laeven
Amsterdam School of Economics
University of Amsterdam, EURANDOM
and CentER
R.J.A.Laeven@uva.nl

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Abstract

In the economics of risk, the primal moments of mean and variance play a central role to define the local index of absolute risk aversion. In this note, we show that in canonical non-EU models dual moments have to be used instead of, or on par with, their primal counterparts to obtain an equivalent index of absolute risk aversion.

Keywords: Risk Premium; Expected Utility; Dual Theory; Rank-Dependent Utility; Local Index of Absolute Risk Aversion.

JEL Classification: D81.

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Eeckhoudt: Catholic University of Lille, IÉSEG School of Management, 3 Rue de la Digue, Lille 59000, France. Laeven: University of Amsterdam, Amsterdam School of Economics, PO Box 15867, 1001 NJ Amsterdam, The Netherlands.

1 Introduction

In their important seminal work, Arrow [1, 2] and Pratt [12] (henceforth, AP) show that under expected utility (EU) the risk premium π attached to a small zero-mean risk $\tilde{\varepsilon}$ is approximated by

$$\pi \simeq \frac{\mathtt{m}_2}{2}\left(-\frac{U''(w_0)}{U'(w_0)}\right), \tag{1.1}$$

where $\mathtt{m}_2$ is the second moment about the mean (i.e., the variance) of $\tilde{\varepsilon}$ while $U'(w_0)$ and $U''(w_0)$ are the first and second derivatives of the utility function of wealth U at the initial wealth level w_0.[1,2]

This well-known result has led to many developments and applications within the EU model. The aim of this note is to show that a similar result can be achieved outside EU, in the dual theory of choice under risk (DT; Yaari [18]) and, more generally, under rank-dependent utility (RDU; Quiggin [13]), provided that the primal second moment $\mathtt{m}_2$ is substituted or complemented by its dual counterpart.[3,4] This modification allows us to develop for the two non-EU models a local index of risk attitude that resembles the one in (1.1) under EU.[5,6]

Our note is organized as follows. In Section 2 we define the second dual moment and we use it in Section 3 to develop the local index of absolute risk aversion under DT. In Section 4 we extend our results to the RDU model. In Section 5 we present an application to portfolio choice and we provide a conclusion in Section 6.

2 The Second Dual Moment

The second dual moment about the mean of a risk $\tilde{\varepsilon}$, denoted by $\bar{\mathtt{m}}_2$, is defined by

$$\bar{\mathtt{m}}_2 := \mathbb{E}\left[\max\left(\tilde{\varepsilon}^{(1)}, \tilde{\varepsilon}^{(2)}\right)\right] - \mathbb{E}\left[\tilde{\varepsilon}\right], \tag{2.1}$$

[1] In the AP approach, the designation "small" refers to a risk that has a probability mass equal to unity but a small variance.

[2] For ease of exposition, we assume U to be twice continuously differentiable.

[3] Dual moments are sometimes referred to as mean order statistics in the statistics literature; see Section 2 for further details.

[4] The RDU model encompasses both EU and DT as special cases and is at the basis of (cumulative) prospect theory (Tversky and Kahneman [16]).

[5] In a very stimulating strand of research, Chew, Karni and Safra [5] and Roëll [14] have developed the "global" counterparts of the results presented here; see also the more recent Chateauneuf, Cohen and Meilijson [3, 4] and Ryan [15]. Surprisingly, the "local" approach has received no attention under DT and RDU, except—to the best of our knowledge—for a relatively little used paper by Yaari [17]. Specifically, Yaari exploits a uniformly ordered local quotient of derivatives (his Definition 4) with the aim to establish global results, restricting attention to DT. Yaari does not analyze the local behavior of the risk premium nor does he make a reference to dual moments.

[6] The insightful Nau [11] proposes a significant generalization of the AP measure of local risk aversion in another direction. He considers the case in which probabilities may be subjective, utilities may be state-dependent, and probabilities and utilities may be inseparable, by invoking the elementary definition of risk aversion of "payoff convex" preferences, which agrees with the concept of aversion to mean-preserving spreads under EU.

where $\tilde{\varepsilon}^{(1)}$ and $\tilde{\varepsilon}^{(2)}$ are two independent copies of $\tilde{\varepsilon}$. The second dual moment can be interpreted as the expectation of the largest order statistic: it represents the expected best outcome among two independent draws of the risk.[7]

Our analysis will reveal that for an RDU maximizer who evaluates a small zero-mean risk, the second dual moment stands on equal footing with the variance as a fundamental measure of risk. While the variance provides a measure of risk in the "payoff plane", the second dual moment can be thought of as a measure of risk in the "probability plane". Indeed, for a risk $\tilde{\varepsilon}$ with probability distribution F, so[8]

$$\mathtt{m} := \mathbb{E}\left[\tilde{\varepsilon}\right] = \int x \, \mathrm{d}F(x),$$

we have that

$$\mathtt{m}_2 = \int (x - \mathtt{m})^2 \, \mathrm{d}F(x), \qquad \text{while} \qquad \bar{\mathtt{m}}_2 = \int (x - \mathtt{m}) \, \mathrm{d}(F(x))^2.$$

For the sake of brevity and in view of (2.1), we shall term the second dual moment about the mean, $\bar{\mathtt{m}}_2$, the *maxiance* by analogy to the *variance*.[9]

Just like the first and second primal moments occur under EU when the utility function is linear and quadratic, the first and second dual moments correspond to a linear and quadratic probability weighting function under DT. In the stochastic dominance literature, these expectations of order statistics and their higher-order generalizations arise naturally in a perceptive paper by Muliere and Scarsini [10], when defining a sequence of progressive n-th degree "inverse" stochastic dominances by analogy to the conventional stochastic dominance sequence (see Ekern [8] and Fishburn [9]).[10]

3 Local Risk Aversion under the Dual Theory

In order to develop the local index of absolute risk aversion under DT we start from a lottery A given by the following representation:[11,12]

[7] The definition and interpretation of the 2-nd dual moment readily generalize to the n-th order, $n \in \mathbb{N}_{>0}$, by considering n copies.

[8] Formally, our integrals with respect to functions are Riemann-Stieltjes integrals. If the integrator is a distribution function of a discrete (or non-absolutely continuous) risk, or a function thereof, then the Riemann-Stieltjes integral does not in general admit an equivalent expression in the form of an ordinary Riemann integral.

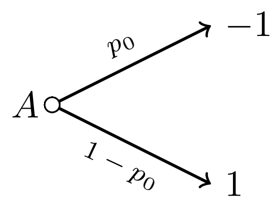
[9] Our designation "small" in "small zero-mean risk" will quite naturally refer to a risk with small maxiance under DT and to a risk with both small variance and small maxiance under RDU.

[10] In a related strand of the literature, Eeckhoudt and Schlesinger [6] and Eeckhoudt, Laeven and Schlesinger [7] derive simple nested classes of lottery pairs to sign the n-th derivative of the utility and probability weighting function, respectively. Their approach can be seen to control the primal moments for EU and the dual moments for DT.

[11] In all figures, values along (at the end of) the arrows represent probabilities (outcomes).

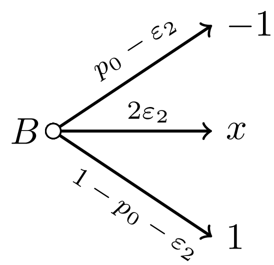
[12] Of course, we assume $0 < p_0 < 1$.

Figure 1: Lottery A



We transform lottery A into a lottery B given by:[13]

Figure 2: Lottery B



To obtain B from A we subtract a probability ε_2 from the probabilities of both states of the world in A without changing the outcomes and we assign these two probabilities jointly, i.e., $2\varepsilon_2$, to a new intermediate state to which we attach an outcome x with $-1 < x < 1$.

If $x \equiv 0$, then $\mathbb{E}[A] = \mathbb{E}[B]$ and B is a mean-preserving contraction of A. Provided the probability weighting (distortion) function $h(p)$ has the usual properties ($h(0) = 0$, $h(1) = 1$, $h'(p) > 0$ and $h''(p) < 0$),[14,15] then the corresponding DT maximizer is averse to mean-preserving spreads and prefers B over A.[16]

To achieve indifference between A and B for such a decision-maker, x has to be smaller than 0 and naturally the difference between 0 and x, denoted by ρ, represents the risk premium

[13]We assume $0 < \varepsilon_2 < \min\{p_0, 1-p_0\}$.

[14]For ease of exposition, we assume h to be twice continuously differentiable.

[15]Rather than distorting "decumulative" probabilities (as in Yaari [18]), we adopt the convention to distort cumulative probabilities. Our convention ensures that aversion to mean-preserving spreads corresponds to $h'' < 0$ (i.e., concavity) under DT, just like it corresponds to $U'' < 0$ under EU, which facilitates the comparison. Should we adopt the convention to distort decumulative probabilities, the equivalent probability weighting function would be convex instead of concave. Furthermore, under that convention, the local index of risk aversion under DT would be linked to the expected *first* order statistic instead of to the expected *largest* order statistic, $\bar{\mathtt{m}}_2$. Of course, for symmetric zero-mean risks, the expected *largest* order statistic equals minus the expected *first* order statistic, and, just like is true for the AP approximation to the risk premium under EU, asymmetries in the distribution of the risk don't get reflected in a second-order approximation to the risk premium under DT.

[16]See the references in footnote 5 for global results on risk aversion under DT and RDU.

attached to the risk change from A to B.

For $x \equiv 0 - \rho$ in B, indifference between A and B implies:

$$\begin{aligned} & h(p_0)(w_0 - 1) + (1 - h(p_0))(w_0 + 1) \\ & = h(p_0 - \varepsilon_2)(w_0 - 1) + (h(p_0 + \varepsilon_2) - h(p_0 - \varepsilon_2))(w_0 - \rho) + (1 - h(p_0 + \varepsilon_2))(w_0 + 1), \end{aligned} \tag{3.1}$$

where w_0 is the decision-maker's initial wealth level. From (3.1) we obtain the explicit solution

$$\rho = \frac{(h(p_0) - h(p_0 - \varepsilon_2)) - (h(p_0 + \varepsilon_2) - h(p_0))}{(h(p_0 + \varepsilon_2) - h(p_0 - \varepsilon_2))}. \tag{3.2}$$

By approximating $h(p_0 \pm \varepsilon_2)$ in (3.2) using second-order Taylor series expansions around $h(p_0)$, we obtain

$$\rho \simeq \frac{\bar{\mathtt{m}}_2}{2\mathtt{Pr}} \left(-\frac{h''(p_0)}{h'(p_0)} \right), \tag{3.3}$$

where $\bar{\mathtt{m}}_2$ is the maxiance of the risk $\tilde{\varepsilon}_2$ that describes the mean-preserving spread from B (with $x \equiv 0$) to A effectively taking the values ± 1 each with probability ε_2, and where $\mathtt{Pr}$ is the total effective probability mass of $\tilde{\varepsilon}_2$.[17]

It is important to compare the result in (3.3) to that obtained by AP presented in (1.1). In AP the local approximation of the risk premium is proportional to the variance, while under DT it is proportional to the maxiance.

We note that the local approximation of the risk premium in (3.3) remains valid in general, for (potentially non-binary) zero-mean risks with small maxiance, just like, as is well-known, (1.1) is valid for (potentially non-binary) zero-mean risks with small variance.[18]

4 Local Risk Aversion under Rank-Dependent Utility

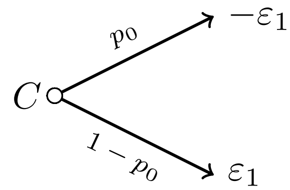
Under DT the local index arises from a risk change with small maxiance. To deal with the RDU model, which includes both EU and DT as special cases, we naturally have to consider changes in risk that are small in both variance and maxiance. To achieve this, we start from a lottery C given by:[19]

[17]Observe that lottery A is obtained from lottery B (with $x \equiv 0$) by attaching the risk $\tilde{\varepsilon}_2$ to the intermediate branch of B. That is, the risk $\tilde{\varepsilon}_2$ is "effective" only in the intermediate state of lottery B, which occurs with probability $2\varepsilon_2$, while it is "ineffective" when lottery B yields outcomes -1 or 1. Of course, the same applies to independent drawings of $\tilde{\varepsilon}_2$. One readily verifies that, for this risk $\tilde{\varepsilon}_2$, we have $\bar{\mathtt{m}}_2 = 2\varepsilon_2^2$ and $\mathtt{Pr} = 2\varepsilon_2$. We consider the maxiance of the zero-mean risk $\tilde{\varepsilon}_2$ to be "small" and, accordingly, compute the Taylor expansions up to the order ε_2^2. The local approximation of the risk premium ρ appears to be proportional to ε_2.

[18]Detailed derivations are suppressed to save space. They are contained in supplementary material available from the authors' webpages.

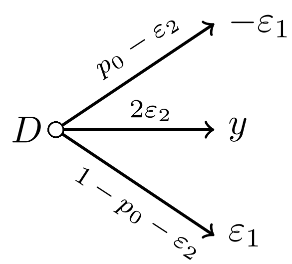
[19]We assume $\varepsilon_1 > 0$.

Figure 3: Lottery C



Similar to under DT, we transform lottery C into a lottery D by reducing the probabilities of both states in C by a probability ε_2 and assigning the released probability $2\varepsilon_2$ to an intermediate state with outcome y, where $-\varepsilon_1 < y < \varepsilon_1$. This yields a lottery D given by:

Figure 4: Lottery D



Of course, when $y \equiv 0$, D is a mean-preserving contraction of C. All RDU decision-makers that are averse to mean-preserving spreads therefore prefer D over C in that case. Thus we can search for $y < 0$ such that indifference between C and D occurs.

The discrepancy between the resulting y and 0 is the RDU risk premium associated to the risk change from C to D and its value, denoted by λ, is the solution to

$$\begin{aligned} & h(p_0)\, U(w_0-\varepsilon_1) + (1-h(p_0))\, U(w_0+\varepsilon_1) \\ = & \; h(p_0-\varepsilon_2)\, U(w_0-\varepsilon_1) + (h(p_0+\varepsilon_2) - h(p_0-\varepsilon_2))\, U(w_0-\lambda) + (1-h(p_0+\varepsilon_2))\, U(w_0+\varepsilon_1)\,. \end{aligned} \tag{4.1}$$

Approximating the solution to (4.1) by Taylor series expansions[20] we obtain

$$\lambda \simeq \frac{\mathtt{m}_2}{2\mathtt{Pr}}\left(-\frac{U''(w_0)}{U'(w_0)}\right) + \frac{\bar{\mathtt{m}}_2}{2\mathtt{Pr}}\left(-\frac{h''(p_0)}{h'(p_0)}\right), \tag{4.2}$$

where $\mathtt{m}_2$ and $\bar{\mathtt{m}}_2$ are the variance and maxiance of the risk $\tilde{\varepsilon}_{12}$ that dictates the mean-preserving

[20] Up to the first order in λ around $U(w_0)$ and up to the second orders in ε_1 and ε_2 around $U(w_0)$ and $h(p_0)$.

spread from D (with $y \equiv 0$) to C effectively taking the values $\pm\varepsilon_1$ each with probability ε_2, and where $\texttt{Pr}$ is the total effective probability mass of $\tilde{\varepsilon}_{12}$.[21]

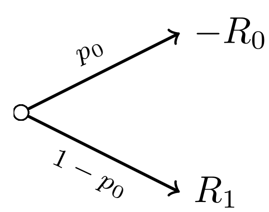
Comparing (4.2) to (1.1) and (3.3) reveals that the local approximation of the RDU risk premium aggregates the (suitably scaled) EU and DT counterparts, with the variance and maxiance featuring equally prominently.

As shown in online supplementary material, the local approximation of the RDU risk premium in (4.2) also generalizes naturally to non-binary risks.

5 A Portfolio Application

In order to illustrate how the concepts we have developed can be used we consider a simple portfolio problem with a safe asset, the return of which is zero, and a binary risky asset with returns expressed by the following representation:[22]

Figure 5: Return Distribution of the Risky Asset



Taking $\frac{R_1}{R_0+R_1} > p_0$ makes the expected return strictly positive.

If an RDU investor has initial wealth w_0 his optimization problem is given by

$$\arg\max_\alpha \left\{ h\left(p_0\right) U\left(w_0 - \alpha R_0\right) + \left(1 - h\left(p_0\right)\right) U\left(w_0 + \alpha R_1\right) \right\}, \tag{5.1}$$

with first-order condition (FOC) given by

$$-R_0 h\left(p_0\right) U'\left(w_0 - \alpha R_0\right) + R_1 \left(1 - h\left(p_0\right)\right) U'\left(w_0 + \alpha R_1\right) \equiv 0.$$

It is straightforward to show that the second-order condition for a maximum is satisfied provided $U'' < 0$.

Let us now pay attention to the RDU investor for whom it is optimal to choose not to invest in the risky asset, i.e., to select $\alpha \equiv 0$. Plugging $\alpha \equiv 0$ into the FOC we obtain the condition

$$h\left(p_0\right) \equiv \frac{R_1}{R_0 + R_1}. \tag{5.2}$$

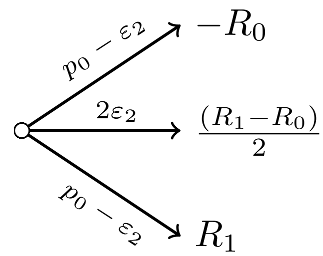
[21]It is straightforward to verify that, for $\tilde{\varepsilon}_{12}$, we have $\texttt{m}_2 = 2\varepsilon_1^2\varepsilon_2$, $\bar{\texttt{m}}_2 = 2\varepsilon_1\varepsilon_2^2$, and $\texttt{Pr} = 2\varepsilon_2$.

[22]We assume $0 < R_0 < R_1$.

Without surprise, $h(p_0) > p_0$. This value of $h(p_0)$ expresses the intensity of risk aversion that induces the choice of $\alpha \equiv 0$.

Now consider a mean-preserving contraction of the return of the risky asset given by:

Figure 6: Mean-Preserving Contraction of the Risky Asset



One may verify that such a mean-preserving contraction for a decision-maker who had decided not to participate in the risky asset will induce him to select a strictly positive α.

Hence, we raise the following question: By how much should we reduce the intermediate return $\frac{R_1 - R_0}{2}$ to induce the decision-maker to stick to the optimal α equal to zero? The answer to this question is denoted by ς.

Because we are concentrating on the situation where $\alpha \equiv 0$ is optimal, the analysis is related only to the concavity of the probability weighting function. Indeed, the concavity of U that appears in the FOC through different values of U' becomes irrelevant at $\alpha \equiv 0$. The reason to concentrate on $\alpha \equiv 0$ where only the probability weighting function matters under RDU pertains to the well-known fact that under EU a mean-preserving contraction of the risky return has an ambiguous effect on the optimal α.

It turns out that, upon invoking Taylor series expansions and after several basic manipulations, the reduction ς that answers our question raised above is given by

$$\varsigma \simeq \frac{\bar{\mathtt{m}}_2}{2\mathtt{Pr}} \left(-\frac{h''(p_0)}{h'(p_0)} \right), \tag{5.3}$$

where $\bar{\mathtt{m}}_2$ is the maxiance of the risk that takes the values $\pm\frac{R_0+R_1}{2}$ each with probability ε_2, and where $\mathtt{Pr}$ is the total probability mass of this risk. Again the second dual moment (instead of the primal one) appears, jointly with the intensity of risk aversion induced by the probability weighting function.

6 Conclusion

Under EU, the risk premium is approximated by an expression that multiplies half the variance of the risk (i.e., its primal second central moment) by the local index of absolute risk aversion.

This expression dissects the complex interplay between the risk's probability distribution, the decision-maker's preferences and his initial wealth that the risk premium in general depends on. Surprisingly, a similar expression almost never appears in non-EU models.

In this note, we have shown that when one refers to the second dual moment—instead of, or on par with, the primal one—one obtains quite naturally an approximation of the risk premium in canonical non-EU models that mimics the one obtained within the EU model.

The approximation of the risk premium under EU has induced thousands of applications and results in many fields such as finance, insurance, and environmental economics. So far, comparable developments have been witnessed to a much lesser extent outside the EU model. Hopefully, the new and simple expression of the approximated risk premium may contribute to a widespread analysis and use of risk premia for non-EU.

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SUPPLEMENTARY MATERIAL (TO BE PUBLISHED ONLINE)

A Generalization to Non-Binary Risks

In this supplementary material, we first show that the local approximation of the risk premium in (3.3) remains valid for non-binary risks with small maxiance. Next, we prove that (4.2) also remains valid for non-binary risks with both small variance and small maxiance. Throughout this supplement, we consider n-state risks with probabilities p_i associated to outcomes x_i, $i = 1, \dots, n$, with $n \in \mathbb{N}_{>0}$. We order states from the lowest outcome state (designated by state number 1) to the highest outcome state (designated by state number n), which means that $x_1 \leq \cdots \leq x_n$.

We analyze the DT risk premium for a risk with $n \geq 2$ effective states that have equal probability of occurrence given by $\frac{2\varepsilon_2}{n}$, $0 < \varepsilon_2 \leq \frac{1}{2}$. The outcomes are, however, allowed to be the same among adjacent states; this would correspond to a risk with non-equal state probabilities. Note the generality provided by this construction. We suppose that the risk has mean equal to zero, so $\sum_{i=1}^{n} x_i = 0$. One may verify that the maxiance of this n-state risk is given by

$$\bar{\mathtt{m}}_2 = \frac{4\varepsilon_2^2}{n^2} \sum_{i=1}^{n} (2i-1)\, x_i, \tag{A.1}$$

and that the total probability mass $\mathtt{Pr} = 2\varepsilon_2$. Observe that the maxiance is of the order ε_2^2, i.e., $\bar{\mathtt{m}}_2 = O\left(\varepsilon_2^2\right)$.

Similar to the main text, this zero-mean risk is attached to the intermediate branch of lottery B (with $x \equiv 0$) to induce a mean-preserving spread. (We normalize the outcomes of the zero-mean risk by restricting them to the interval $[-1, 1]$. This ensures that the initial ordering of outcomes in lottery B is not affected and can easily be generalized.) The DT risk premium ρ then occurs as the solution to

$$\begin{aligned}
&\left(h\left(p_0+\varepsilon_2\right)-h\left(p_0-\varepsilon_2\right)\right)\left(w_0-\rho\right) \\
&= \sum_{i=1}^{n}\left(h\left(p_0-\varepsilon_2+i\frac{2\varepsilon_2}{n}\right)-h\left(p_0-\varepsilon_2+(i-1)\,\frac{2\varepsilon_2}{n}\right)\right)\left(w_0+x_i\right).
\end{aligned} \tag{A.2}$$

From (A.2) we obtain the explicit solution

$$\rho = -\sum_{i=1}^{n} \frac{\left(h\left(p_0-\varepsilon_2+i\frac{2\varepsilon_2}{n}\right)-h\left(p_0-\varepsilon_2+(i-1)\,\frac{2\varepsilon_2}{n}\right)\right)}{h\left(p_0+\varepsilon_2\right)-h\left(p_0-\varepsilon_2\right)} x_i. \tag{A.3}$$

By invoking Taylor series expansions around $h\left(p_0\right)$ up to the second order in ε_2 we obtain from (A.3) the approximation

$$\rho \simeq -\sum_{i=1}^{n} \frac{\frac{1}{2}(2i-1)\frac{4\varepsilon_2^2}{n^2}h''(p_0)}{2\varepsilon_2 h'(p_0)} x_i$$
$$= \frac{\bar{\mathtt{m}}_2}{2\mathtt{Pr}}\left(-\frac{h''(p_0)}{h'(p_0)}\right),$$

as desired, where the last equality follows directly from (A.1).

Finally, turning to the risk premium under RDU, we consider, as under DT, an n-state zero-mean risk with state probabilities $\frac{2\varepsilon_2}{n}$, so $\sum_{i=1}^{n} x_i = 0$ and $\mathtt{Pr} = 2\varepsilon_2$, now assumed to satisfy additionally that $\mathtt{m}_2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 = O\left(\varepsilon_1^2\right)$ for some $\varepsilon_1 > 0$. Upon attaching this zero-mean risk to the intermediate branch of lottery D (with $y \equiv 0$ and assuming without losing generality that $|x_i| < \varepsilon_1$), the RDU risk premium λ occurs as the solution to

$$\begin{aligned}&\left(h\left(p_0+\varepsilon_2\right)-h\left(p_0-\varepsilon_2\right)\right)U\left(w_0-\lambda\right)\\&=\sum_{i=1}^{n}\left(h\left(p_0-\varepsilon_2+i\frac{2\varepsilon_2}{n}\right)-h\left(p_0-\varepsilon_2+(i-1)\frac{2\varepsilon_2}{n}\right)\right)U\left(w_0+x_i\right).\end{aligned} \tag{A.4}$$

Invoking Taylor series expansions up to the first order in λ around $U(w_0)$ and up to the second order in x_i and ε_2 around $U(w_0)$ and $h(p_0)$, we obtain from (A.4), at the leading orders, the desired approximation

$$\lambda \simeq -\sum_{i=1}^{n} \frac{\frac{1}{2}\frac{2\varepsilon_2}{n}U''(w_0)}{2\varepsilon_2 U'(w_0)} x_i^2 - \sum_{i=1}^{n} \frac{\frac{1}{2}(2i-1)\frac{4\varepsilon_2^2}{n^2}h''(p_0)}{2\varepsilon_2 h'(p_0)} x_i$$
$$= \frac{\mathtt{m}_2}{2\mathtt{Pr}}\left(-\frac{U''(w_0)}{U'(w_0)}\right) + \frac{\bar{\mathtt{m}}_2}{2\mathtt{Pr}}\left(-\frac{h''(p_0)}{h'(p_0)}\right).$$