How does the regulatory valuation regime affect life insurance investment behaviour?

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ABSTRACT

This paper examines how the interactions between the valuation regime and solvency requirements influence insurers' investment behaviour when they have incentives to risk-shift to policyholders under limited liability. Solvency requirements based on historical cost valuation encourage risk-shifting, while those based on fair value regime can lead to procyclical asset sales. A hybrid valuation regime, which has been intended by policymakers to avoid these unfavourable outcomes, does not strictly dominate the other regimes. Instead, the socially optimal outcome can be achieved by supplementing the fair value regime with regulatory flexibility to adjust the penalty imposed on those insurers that breach the solvency constraint only due to temporary asset price falls.

I. Introduction

The distinct features of the business of life insurance present a number of challenges in designing an appropriate valuation framework[1] for solvency regulation. The debt-like liabilities of life insurers, such as annuities with fixed guarantees, are typically long-term in nature, and hence are not normally subject to 'runs' in the same way that bank deposits are. This has two conflicting implications. On the one hand, stable liabilities make life insurers less prone than banks to engage in asset fire sales, which could generate a damaging cycle of asset price falls followed by further asset sales. On the other hand, the fact that the main claimholders of life insurers consist mostly of policyholders, who cannot easily surrender their insurance policies and are often dispersed and insufficiently informed, implies that that their management might not be subject to the kind of creditor disciplines that banks are under. Thus, life insurers may have the ability to conceal the losses on their assets for a long time if the valuation regime allows it, as they try to 'gamble for resurrection' which could magnify the losses for policyholders when they ultimately fail.

The valuation regime used to measure insurers solvency can deeply affect their behaviour. For instance, measuring assets at fair value can in principle reduce such risk-shifting incentives. However, this might come at the cost of generating some procyclical investment behaviour. Mitigating these incentives through the valuation regime is an extremely challenging task. In this paper we aim at addressing this problem, by analysing in a simple theoretical framework how valuation regimes, and their interactions with solvency requirements, affect life insurers investment behaviour.

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[1] Whilst we use the words valuation and accounting interchangeably in the paper, we are referring throughout to valuation for solvency purposes rather than for the general purpose of financial reporting.

The debate over how to measure the assets and liabilities of life insurers has swung like a pendulum throughout the history. In the United Kingdom, the 1870 Life Assurance Companies Act mandated life insurers to produce periodic actuarial valuations of their assets and liabilities, but it allowed discretion in the valuation of assets, so that the use of market prices was common but not universal[2]. From the 1990s on, however, there was growing support for making greater use of market prices in valuing assets and liabilities of insurers, for two main reasons. First, there was a general movement in accounting standard-setting to enhance transparency by giving greater emphasis to the use of fair values. In the United States, the issue of SFAS 115 in 1993 required firms to measure securities that were not held to maturity at fair value. International accounting standards moved in the same direction: the issue of the accounting standards IAS 39 in 1998 followed by IAS 40 in 2000 increased the use of fair value accounting in financial instruments and investment properties, respectively. Second, there were a number of high-profile failures of life insurers in several parts of the world from the 1990s onwards, including Executive Life Insurance Company (ELIC) in the United States, multiple life insurers in Japan, and Europavie in France, as the setting of guaranteed benefits to policyholders at high levels left them exposed to interest rate risks. Importantly, the perceived failure of life insurers to act promptly on the problem once it had been identified strengthened the case for fair value accounting in some jurisdictions, on the ground that it would have made the build-up of risks more visible and thus would have encouraged corrective actions earlier. In the United Kingdom, 'realistic' financial reporting requirements, based on market-consistency and including expected future bonuses as a liability, were mandated for insurers following the near-collapse of Equitable Life Assurance Society, which was forced to close to new business in 2000.

But the flaws of the fair value (FV) regime were thrown into sharp relief during downturns, including the 2007-9 global financial crisis. As asset prices fell across global markets, financial institutions that were forced to mark their assets to market dumped assets to reduce leverage, thus causing asset prices to spiral down further. Insurance regulators across the world responded to the financial crisis by exercising discretion. For example, the falls in asset prices in 2007-9 could have caused certain UK insurers to breach their Individual Capital Guidance (ICG) set by the Financial Services Authority (FSA), which was responsible for financial regulations at that time. But the FSA, concerned that insurers might offload assets into a falling market, relaxed its approach to breaches of ICG, as documented in a letter from Paul Sharma to Angela Knight[3]. The FSA stated that economic circumstances were "exceptional" and that "very occasionally" it was possible that a breach due to extraordinary market conditions would not lead to automatic regulatory action. US agencies also took action in the wake of the 2008 crisis. In some states, statutory accounting practices set by the insurance regulator were revised in a way that allowed insurers to improve their capital positions. Both Life and Property & Casualty insurers benefitted from an improvement in capital position in the years following 2008 as a result. In addition, following rating agencies' downgrade of non-agency residential mortgage-backed securities (RMBS) that would require increased capital charges for these assets, the National Association of Insurance Commissioners (NAIC) took the view that ratings had become overly pessimistic. As a result, a new methodology based on expected recovery values was introduced that had the effect of reducing capital requirements for 59% of the insurance industry's non-agency RMBS investments [13]. Regulatory changes as a result of extraordinary market conditions, whether to the valuation regime or some other part of the framework, were also made in Denmark, the Netherlands, Sweden and Switzerland [4].

[2]We refer to [9] for a discussion on the development of life assurance accounting and regulation in the UK.

[3]Financial Services Authority 2009, 'ICG and GENPRU 1.2.26R'. The letter is available at `http://www.fsa.gov.uk/static/pubs/other/letter_to_bba.pdf`

The 2007-9 financial crisis also gave rise to the new academic literature, which questioned the desirability of fair value accounting. For example, [1] argued that fair value accounting can lead to contagion between the banking and insurance sectors, as it induces insurance companies to sell their assets in response to a shock, and the resulting fall in asset prices can make otherwise healthy banks insolvent. Similarly, [12] developed a theoretical model to demonstrate that FV accounting could generate larger welfare losses than the historical cost (HC) accounting when applied to long-lived, illiquid and senior assets. Yet, their analysis makes a number of assumptions which may not be suitable for considering the behaviour of life insurers. For instance, they assume that the managers of financial institutions simply maximise the accounting values of their assets rather than the shareholder values, and decide on selling assets based on what they *expect* to get in the market, rather than based on observed market prices. Their framework also does not allow an analysis of how the accounting regime interacts with solvency regulation, and does not consider agency problems between shareholders and debtholders. Thus, it does not incorporate the possibility that HC accounting can generate other types of inefficiencies, such as risk-shifting, by allowing financial institutions to conceal losses.

Others, however, expressed doubts that a return to the HC accounting would be desirable, as they identified problems associated with the HC accounting in empirical studies: for example, [7] and [8] found that in the US those insurers that were subject to the HC accounting were more prone to invest in risky assets and less likely to sell the downgraded assets during the global financial crisis, compared to those that were subject to fair value accounting.

The unresolved academic controversies over the pros and cons about the FV versus HC regimes also led to the exploration of hybrid valuation approaches in policy circles, such as the Solvency II's matching adjustment and volatility adjustment, as well as the 'market adjusted valuation' (MAV) approach under consideration by the International Association of Insurance Supervisors (IAIS).

This paper examines how the interactions between the regulatory valuation regime and the solvency capital requirement influence insurers' investment behaviour using a simple three-period model, in which the insurance management maximises shareholder returns. Our model incorporates two key frictions. First, we assume that policyholders cannot monitor insurers' investment decisions due to asymmetric information, which gives rise to agency problems between the policyholders on one hand, and the insurance companies' management and shareholders, on the other. Under limited liability, life insurers are incentivised to take excessive risks at the expense of policyholders by holding on to assets that have suffered a deterioration in credit quality ('gambling for resurrection'), since policyholders cannot monitor the quality of insurers' assets. Second, we assume that market prices can deviate from fundamental asset values, both due to market illiquidity and other 'noisy' factors, such as over-optimism and pessimism by non-insurance holders of assets. In our model, insurers hold common assets which are traded in illiquid markets, such that the sale of assets by one insurer imposes pecuniary externalities on other insurers by lowering the price at which others can sell their assets. We characterise 'asset fire sales' as a situation in which insurers sell their assets even when their market prices are below the levels consistent with the expected return on those assets.

We use the model to examine how insurers might behave in response to a credit downgrade of their assets under the three alternative valuation regimes: historical cost (HC), fair value (FV) and the hybrid regime. Our analysis yields the following results. First, we demonstrate that the HC regime prevents asset fire sales but makes life insurers prone to gambling for resurrection. As a result, insurers are excessively hesitant to sell their assets that have been downgraded, preferring to hold on to them even when the market is pricing them at levels that are higher than the expected return. While this behaviour maximises shareholders' expectected returns under limited liability, this represents risk-shifting to policyholders whose expected losses increase as a result of insurers' refusal to sell their assets. Second, we show that insurers' behaviour under a FV regime depends crucially on the regulatory penalty imposed on those that breach the solvency constraint. At one extreme, if insurers don't expect to be penalised for breaching the solvency constraint, then the FV regime will generate the same 'gambling' behaviour as under the HC regime. By contrast, if insurers expect the regulator to impose a severe penalty for breaching the solvency constraint, then insurers under the FV regime will respond to negative asset price shocks with asset fire sales. Our paper is therefore closely related to the existing literature which focuses on how the valuation regimes affect agency problems ([5]; [11]).

Our key contribution to the existing literature and the ongoing policy debate on the valuation regime for insurers is to demonstrate that a FV regime combined with regulatory discretion could achieve a socially optimal outcome. In particular, we find that a regulator which is perfectly informed can eliminate both inefficiencies – gambling for resurrection and asset fire sales – under the FV accounting by adjusting the regulatory penalty according to their assessment of the fundamental quality of the assets held by insurers that breach the solvency constraint. We also demonstrate that such a regime would encourage efficient investment decisions by insurers *ex-ante*, whereas the HC regime would encourage excessively risky investment decisions. Our analytical results are therefore consistent with the empirical findings of [7] and [8] that insurers under the HC regime tended to invest in riskier asset portfolios than those under the FV regime.

A key policy implication of our paper is that there is a need to leave scope for regulatory discretion under the FV regime, allowing the regulators to reduce the penalty for breaching the solvency requirement when they judge the insurers' asset quality to be fundamentally sound and that the breach of the FV solvency constraint has occurred due to temporary downward pressures (or 'noise') on market prices. By contrast, we find that replacing the FV regime with a hybrid valuation regime, which places only some weight on the market valuation in calculating the solvency ratio, smoothes out some of the extremes of the other two regimes but does not, by itself, entirely solve the two inefficiencies. In fact, the same regulatory discretion would be needed to make a hybrid regime work, for the simple reason that noisy asset prices can still cause insurers to breach the hybrid solvency constraint – albeit less frequently than under the FV regime – even if their asset portfolios are fundamentally sound. Thus, our analysis suggests that the concerns raised against moving towards hybrid valuation regimes while removing any regulatory flexibility are valid, and that the regulators should not expect tweaks in the valuation regime on their own to correct for inefficient behaviour by life insurers.

[12] asked: Is marking-to-market a panacea or Pandora's box? Our answer is that no valuation regime will offer a panacea, but, even in a second best world where market prices can deviate from fundamental values, a FV regime could work well if the regulator is well informed about the insurers' asset quality and can apply judicious discretion over how to treat those insurers that breach the solvency constraint in times of market volatility. By contrast, finding the 'Goldilocks' hybrid regimes which combine some features of the HC and the FV regimes to correct for all inefficiencies in the system without relying on regulatory discretion may not be possible.

The rest of the paper is organized as follows. In the next section, we outline the set-up of the model. In section (III) we analyze insurers' decisions on whether to sell or hold risky assets following an asset downgrade. In particular, we are interested in comparing their behaviour when regulatory solvency

solvency constraint is specified using different valuation methods (HC, FV and hybrid). In section (IV), we study how these valuation regimes affect the ex-ante asset allocation problem of forward-looking insurers that can anticipate solvency pressures. The last section concludes.

II. Model

The model consists of three time periods: $t = 0, 1, 2$. There is a continuum of ex-ante identical insurance companies with unit mass. Insurers are risk-neutral and maximise shareholders' payoff, given by the expected value of assets net of the expected policyholders payments. However, insurers are protected by limited liability, thus they do not internalize policyholders losses. Policyholders cannot monitor insurers' investment risk and cannot exert any discipline on them. This agency problem between policyholders and shareholders gives rise to a rationale for prudential regulation in our model.

At $t = 0$, each insurer has equity equal to E_0, and collects one unit of premium from policyholders in exchange for life insurance contracts[4]. More specifically, each insurance company promises to pay out to policyholders an amount D_1 at $t = 1$ and an amount D_2 at $t = 2$. Throughout the paper we assume that there is no uncertainty about D_1 and D_2. This is because certain types of insurance contracts, such as annuities and life-insurance contracts, have predictable payouts and we restrict our attention to policies which are not surrendable[5]. For example, the insurance contract can be thought as an annuity, paying policyholders that are still alive in both periods. In this case, we have that $D_1 = (1 - d)D$ and $D_2 = (1-d)^2 D$, where d is the death probability and D the annuity payout. Alternatively, a life insurance contract could promise an amount D upon the death of the insured person. In this case we would have $D_1 = dD$ and $D_2 = d(1-d)D$. In the rest of the paper we do not specify the form of insurance payments.

At $t = 0$ the insurer can invest the policy premium and its own equity, $A_0 = 1 + E_0$, in two types of securities: a short-term safe asset (e.g. government bond) yielding unitary return at $t = 1$, and a long-term risky asset (e.g. corporate bond) which matures at $t = 2$. For simplicity, we assume that the risky asset yields $R > 1$ if there is no default and zero in case of default. The risky asset can also be subject to a credit rating downgrade at $t = 1$. If there is no downgrade at $t = 1$, the asset yields R at $t = 2$ with probability one[6], while if there is a downgrade, it is expected to yield R only with probability $0 < z_1 < 1$. We assume that, as at $t = 0$, the probability of a downgrade (at $t = 1$) is $0 \leq p \leq 1$, so the expected return of the risky asset is given by $[(1 - p) + pz_1]R$. Insurers can acquire both the risky and safe assets at unitary value at $t = 0$, but we assume that the risky asset yields a higher expected return, that is $[(1 - p) + pz_1]R > 1$.

At $t = 0$, insurers need to invest enough in the safe asset to be able to pay out D_1 to policyholders at t=1. Thus, in our model insurers have short-term liabilities that are perfectly matched with their short-term assets. For the moment, we take as given that every insurer invests a fraction b of its assets in the short-term safe security at $t = 0$, which is just sufficient to meet the $t = 1$ insurance payout:

$$b = \hat{b} \equiv \frac{D_1}{A_0}. \tag{II.1}$$

[4]Note in our model insurance contracts' supply and demand are taken as given and we do not endogenize insurance price.

[5]This is true of some UK contracts, such as annuities. Moreover, other contracts may have significant surrender penalties or tax implications that deter surrender.

[6]Note, while this assumption has been made for simplicity, making the asset's payoff risky even in the absence of a downgrade would not change the main results of the paper.

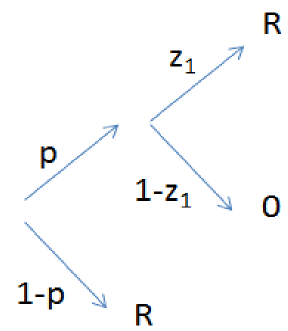


Figure 1. Payoff structure of the risky asset as at $t = 0$.

Insurers invest the remaining fraction $1 - b$ in the risky asset. Note that this is indeed the optimal asset allocation for insurers that do not anticipate the possibility of a downgrade and any associated solvency pressures. In Section IV we will analyze the optimal initial asset allocation of forward-looking insurers and provide conditions under which this asset allocation continue to remain optimal. In order to set up the insurance company at $t = 0$, insurers need to satisfy the solvency constraint:

$$\frac{\text{Liabilities}}{\text{Assets under stress}} = \frac{D_1 + D_2}{bA_0 + c(1-b)A_0} \leq 1 \tag{II.2}$$

where $0 < c \leq 1$ represents the asset mark-down in stress, i.e. one minus the percentage change in the asset value calculated following a one-in-200-year shock[7]. The lower c, the tighter the regulatory capital requirement on risky assets relative to safe assets. We refer to the Appendix for a formal derivation of this constraint. Note that the above constraint is satisfied as long as insurers have enough initial equity:

$$E_0 \geq \frac{cD_1 + D_2}{c} - 1. \tag{II.3}$$

At $t = 1$ insurers observe whether the risky asset is downgraded or not, and decide whether to sell the asset at the prevailing market price, or to hold it until maturity at $t = 2$. In the latter case, insurers are able to pay out policyholders at $t = 2$ only if the asset does not default. We assume that in that case the payoff of the risky asset is large enough to cover the policyholders' payout entirely

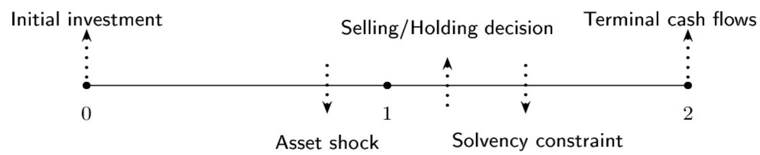
$$z_1 R \geq \frac{D_2}{A_0 - D_1} \equiv X \tag{II.4}$$

For simplicity, we assume that, if there is no downgrade, the market price of the risky asset will be fair at R. In this case, since insurers do no have incentives to sell, they simply hold the asset till maturity and pay back their policyholders the full amount that is due[8]. Thus, in the rest of this section we will focus on the more interesting case of the asset being downgraded.

If the risky asset is downgraded at $t = 1$, insurers will choose whether to sell it and obtain cash or hold it to maturity. We assume that insurers which want to sell form a queue, and the place of a given

[7]Since liabilities are fixed they do not change value under stress.

[8]To be precise insurers are indifferent between selling and holding the asset when there is no downgrade. In either case their payoff is $R - D_2$, that is the asset payoff net of the policyholders payments.



insurer in the queue is uniformly distributed in the interval $[0, \hat{s}]$ where $\hat{s}$ is the total fraction of insurers selling. In particular, insurers sell *sequentially*, and the s-th seller decides whether to sell or hold when its turn comes, depending on the price observed in the market.

This assumption contrasts with that made by [12], who assumed that financial institutions decide *simultaneously* whether to sell or not depending on the *expected* return from selling versus holding[9]. In essence, financial institutions in [12] decide on trading before they observe the price which they can obtain. We think that this modelling strategy is more suitable for the analysis of banks, which face the possibility of a depositor run and therefore may have to act on the asset side in order to avoid that possibility. By contrast, life insurers' liabilities tend to be stable, and hence it is more appropriate to assume that they decide whether to trade or not depending on the actual price they can obtain by selling their assets.

After the downgrade, the market for downgraded assets becomes noisy and illiquid. Therefore, the market price will not only reflect the asset fundamental value, given by z_1R, but also some short-term volatility. Throughout we assume insurers can observe z_1. The price that the s-th seller can obtain for selling the downgraded risky asset at $t = 1$ is given by:

$$v_1(s) = \max(z_1R + \varepsilon - \gamma s, 0) \tag{II.5}$$

where γ is the measure of market liquidity. The higher γ, the more illiquid the market, as the marginal sales by the insurers will have a larger negative impact on the market price. The term γs captures the pecuniary externalities that asset sales by one insurer imposes on others that hold similar assets. Note that (II.5) is the standard linear price impact function, as adopted in [12], with a floor of zero to ensure that prices are non-negative. In addition to market liquidity, the price differs from the fundamental value of the asset due to market noise, denoted by ε. This captures the feature that credit and liquidity spreads do not account for the entire bond spread.

It has been well documented that changes in market prices, particularly for fixed income securities, generally exceed changes that could be attributed to changes in credit risk. For instance, [14] have decomposed corporate bond spreads using a structural credit risk model and have shown that compensation for bearing non-credit related illiquidity risk have been a particularly important driver, particularly during the recent financial crisis. Furthermore, even after accounting for a liquidity premium, the residual component can be quite sizable, as shown in Figure 2 for sterling investment grade corporate bonds.

The residual can be interpreted as market noise, due to market participants' aggregate perceptions and confidence about the value of the assets and the financial health of the issuer. The mispricing could also arise because of their risk-preferences or their need to trade for other reasons, and because the presence of uninformed traders. Therefore, ε represent the extent to which the security is over- or undervalued. In the rest of the paper we interpret it as market pessimism or optimism, although we do not take a stand

[9]Specifically in their model the expected return depends on the noisy private signal they receive about the fundamental, and the trading strategies of other players that they infer from that noisy signal.

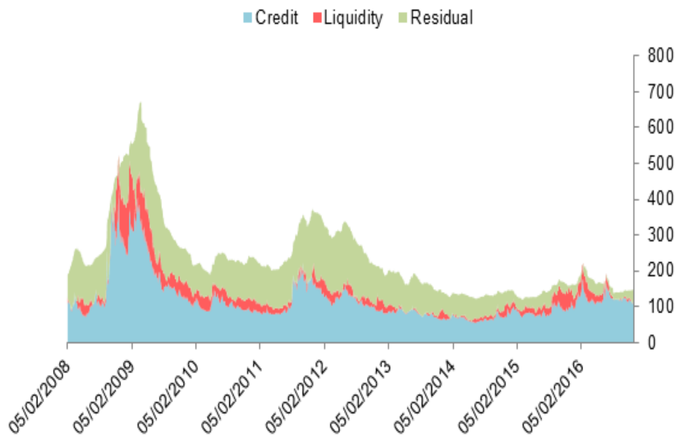


Figure 2. Estimates of the spread for credit risk, liquidity risk and noise for Sterling investment grade corporate bonds. Data source: Bank of England and JP Morgan DataQuery.

on what drives it. In particular, a negative value $\varepsilon < 0$ indicates excessive market pessimism, assets are undervalued, whereas $\varepsilon > 0$ indicates over-optimism, assets are overvalued. We assume that the market noise is uniformly distributed $\varepsilon \sim \mathcal{U}[\varepsilon^L, \varepsilon^H]$, where $\varepsilon^L < 0$ and $\varepsilon^H < (1 - z_1)R$. This last assumption implies that the market cannot overvalue the asset so much that the market price after the downgrade is higher than the asset's return in case of no downgrade. We denote the density functions of ε with $f(\varepsilon) = \frac{1}{\varepsilon^H - \varepsilon^L}$.

In the next section we examine insurers' decision to sell or hold the risky asset after a downgrade, under different valuation regimes. In Section IV we will then study the ex-ante asset allocation choice between a safe and risky strategy, given that insurers anticipate the possibility of a downgrade.

III. Interim (dis-)investment decision

In the interim period, insurers observe whether the risky asset has been downgraded or not. Given this information, insurers can decide whether to keep the asset until maturity or to sell it at the prevailing market price. When insurers decide to hold the asset to maturity, they can realize a strictly positive profit only if the asset does not default, where default happens with probability z_1. Then, in the absence of regulation, their expected profits from holding are:

$$z_1(RA_0(1 - b) - D_2). \tag{III.1}$$

With probability $1 - z_1$ the asset defaults at $t = 2$ and policyholders lose entirely their payouts. By contrast, if insurers sell their assets they receive cash to pay back policyholders. In particular, their payoff is given by:

$$[v_1(s)A_0(1 - b) - D_2]^+. \tag{III.2}$$

Given limited liability, the lowest payoff that the sellers can end up with is zero, i.e. the insurance company is wound down.

Under limited liability, insurers have incentives to 'risk-shift' to policyholders by taking excessive risks in the interim to benefit shareholders at the expense of policyholders. In particular, in our model shareholders risk-shift by refusing to sell downgraded assets, rather than taking on risky projects as in the traditional models by [10]. The risk-shifting problem we study is similar in spirit to the one analyzed in [6], in which a bank's manager avoids selling illiquid assets, even though such sales could save the bank. The intuition is that, as in our setting, by selling assets the manager would raise cash and makes the bank' s debt safer, but would also sacrifice the returns in case the asset recovers. However, while [6] focus on liquidity shocks caused by depositors' withdrawals, we are interested in a credit shock due to an asset downgrade. This is because we consider insurers' liabilities to be more stable and therefore illiquidity represents only a second-order issue relative to credit risk.

In order to reduce the problem of risk-shifting, the regulator imposes a solvency constraint in the interim period for those insurers who decide to hold the risky assets. Those that decide to sell can continue to operate as long as the sales proceeds exceed the amount due to policyholders at $t = 2$, but are closed down otherwise. Specifically, the $t = 1$ solvency constraint is:

$$\frac{D_2}{cV(A_0(1-b))} \leq 1 \tag{III.3}$$

that is the insurers' liabilities, i.e. policyholders' payout D_2 due at $t = 2$, must be lower than the value under stress of the remaining assets invested in the risky security, denoted by $V(A_0(1-b))$. The asset value will depend on the valuation regime adopted. In what follows, we consider the solvency regulation under three different valuation regimes: i) historical cost (HC), ii) fair value (FV), and iii) market-adjusted valuation (MAV).

Insurers will be subject to a penalty if they breach the solvency constraint at $t = 1$. We assume that the penalty is proportional to the size of their assets, $\rho * A_0(1-b)$, where $\rho > 0$. More concretely, ρ can be interpreted as the cost of the mandatory recapitalization or the cost of restrictions on new business ordered by the regulator. This can include the cost due to dilution of claims by the existing shareholders and the length of time given to recapitalize, or the effect on the value of the franchise arising from the restrictions on new business. In general, we think of ρ as a measure of the regulator's tolerance for breaches of the solvency requirement. In the following, we examine how the regulatory parameters ρ and c affect the behaviour of insurers under different valuation regimes.

A. The socially optimal solution

Before analysing different valuation regimes, we first examine the socially optimal equilibrium which maximises the total expected payoffs of shareholders and policyholders in the insurance industry[10]. The analysis in this section provides a benchmark against which welfare implications of different valuation regimes can be assessed.

The expected return of shareholders at $t = 1$ if they hold the asset until maturity is given by (III.1). In this case, policyholders receive their final payouts only in case of no default, hence their expected payoff is $z_1 D_2$. By contrast, when insurers sell the assets, their shareholders' payoff is given by (III.2) and their policyholders receive:

$$\min(D_2, (1-b)A_0 v_1(s)), \tag{III.4}$$

that is, the policyholders receive entire amount due at $t = 2$ only if the sales proceeds exceed D_2. As all returns from assets accrue to policyholders and shareholders, the social welfare – given by the sum of

[10] Note that insurers' investment behaviour, such as fire sales, can be associated with negative externalities on other sectors. This is for instance accounted for by [1], where a shock to the insurance sector propagates to the banking sector under fair value accounting. In this paper we do not model such externalities and focus our attention on the insurance sector only.

payoffs of policyholders and shareholders – is simply captured by the expected value of insurers' assets, which is equal to $A_0(1-b)z_1R$ when the assets are held to maturity, and is equal to their market value $A_0(1-b)v_1(s)$ when sold. Thus, a social planner will allow insurers to hold the asset to maturity, given that a fraction s sold, only if the expected return on that asset is higher than the market value:

$$z_1R \geq v_1(s), \tag{III.5}$$

Proposition III.1. ***(Socially Optimal Equilibrium)***
The socially optimal level of sales is given by:

$$s^* = \begin{cases} 0 & \textit{if } \varepsilon < 0 \\ \frac{\varepsilon}{\gamma} & \textit{if } 0 \leq \varepsilon \leq \gamma \\ 1 & \textit{if } \varepsilon > \gamma \end{cases} \tag{III.6}$$

and the market price that prevails after insurers sell is given by:

$$v_1^* = \begin{cases} \max(z_1R+\varepsilon, 0) & \textit{if } \varepsilon < 0 \\ z_1R & \textit{if } 0 \leq \varepsilon \leq \gamma \\ z_1R+\varepsilon-\gamma & \textit{if } \varepsilon > \gamma \end{cases} \tag{III.7}$$

Proof Given the planner's decision rule (III.5) and the asset price (II.5), the socially optimal choice is for no insurer to sell ($s^* = 0$) if $\varepsilon < 0$, as the market is under-valuing the asset in this case, and thus insurers can sell the asset only below its fundamental value. All insurers should sell ($s^* = 1$) if $\varepsilon > \gamma$, i.e. the market is overly optimistic and relatively liquid, such that all the insurers can liquidate their long-term assets at a profit. Finally, only a fraction π^s should sell if $0 \leq \varepsilon \leq \gamma$, that is the market is over-valuing the asset but is relatively illiquid, where in equilibrium:

$$\pi^s = \frac{\varepsilon}{\gamma}. \tag{III.8}$$

Thus, the socially optimal choice is for insurers to behave counter-cyclically, selling when the market is over-exuberant and relatively liquid while holding when market prices are depressed. Importantly, the socially optimal choice is independent of credit risk z_1. This implies that insurers should sell only when the market is underestimating the credit risk, regardless of the level of credit risk itself.

Note that the socially optimal rule is not equivalent to the one that minimizes policyholders losses. While the socially optimal rule never induces sales when they are detrimental for policyholders, it does lead to greater holdings of the risky asset relative to the behaviour that would minimize policyholders losses.

B. Historical cost valuation regime

When the regulator adopts historical cost valuation in specifying the solvency constraint, insurers' assets at $t = 1$ are valued at their book value $A_0(1-b)$ as long as they choose to hold on to them. It follows that, the regulator expects those insurers that decide to hold the long-term assets to maturity to meet the following constraint at $t = 1$:

$$X \leq c \tag{III.9}$$

Note that, any insurer that has satisfied the $t = 0$ constraint (II.2) automatically satisfies the $t = 1$ solvency constraint (III.9). This is because, when assets are measured at their book value, the heightened credit risk due to the downgrade is not reflected in their solvency positions. Therefore, under the historical cost regime, the solvency constraint is never binding in the interim period, and the insurers' decision problem is the same as under no regulation. That is, insurers will prefer to hold the asset, given that a fraction s sold, if

$$\underbrace{z_1(R - X)}_{\text{Payoff when holding}} \geq \underbrace{[v_1(s) - X]^+}_{\text{Payoff when selling}} \tag{III.10}$$

Proposition III.2. ***(Historical Cost Equilibrium)***

Under the historical cost valuation regime the equilibrium fraction of sales is:

$$s^{HC} = \begin{cases} 0 \text{ if } \varepsilon < X(1 - z_1) \\ \pi^{HC} = \frac{1}{\gamma}[\varepsilon - X(1 - z_1)] \text{ if } X(1 - z_1) \leq \varepsilon \leq \gamma + X(1 - z_1) \\ 1 \text{ if } \varepsilon > \gamma + X(1 - z_1) \end{cases} \tag{III.11}$$

and the market price that prevails after insurers sell is given by:

$$v_1^{HC} = \begin{cases} \max(z_1 R + \varepsilon, 0) \text{ if } \varepsilon < X(1 - z_1) \\ z_1 R + \varepsilon - \gamma\pi^{HC} = z_1 R + X(1 - z_1) \text{ if } X(1 - z_1) \leq \varepsilon \leq \gamma + X(1 - z_1) \\ z_1 R + \varepsilon - \gamma \text{ if } \varepsilon > \gamma + X(1 - z_1) \end{cases} \tag{III.12}$$

Historical cost equilibrium results in excessive holding (insufficient sales) if $0 < \varepsilon < \gamma + X(1 - z_1)$.

Proof Given the insurers' decision rule under the HC regime (III.10) and asset price (II.5), s-th insurer will hold as long as $\varepsilon \leq \gamma s + X(1 - z_1)$. Thus, no insurer sells if $\varepsilon < X(1 - z_1)$; $\pi^{HC} = \frac{1}{\gamma}[\varepsilon - X(1 - z_1)]$ sells if $X(1 - z_1) \leq \varepsilon \leq \gamma + X(1 - z_1)$; and all insurers sell if $\varepsilon > \gamma + X(1 - z_1)$: this yields (III.11). Plugging in (III.11) into (II.5), we obtain (III.12). Comparing (III.11) with (III.6), it is clear that $s^{HC} < s^*$ if $0 < \varepsilon < \gamma + X(1 - z_1)$.

Note that under the historical cost valuation regime, insurers will sell their assets in the interim only when market conditions are relatively favourable and they can achieve a price above the fundamental value. Further, for given market conditions, insurers with high liabilities-to-assets ratios are the ones that prefer to excessively hold on to the risky assets rather than liquidating. This is in line with the standard risk-shifting motives: due to the implicit protection provided by limited liability, highly leveraged insurers prefer to hold on the risky asset and bet on its recovery, rather than liquidating and securing cash to pay the policyholders.

Proposition (III.2) shows that solvency requirements based on historical cost valuation lead to sub-optimal level of asset sales by insurers after assets are downgraded. This in turn leads to overinflated asset prices when the market is over-optimistic. Note that holding the asset after the downgrade is particularly detrimental when credit risk is high, while it is less inefficient when credit risk remains low. In other words, the range of inefficient excessive holding is reduced the higher z_1 (i.e. lower credit risk).

Proposition III.3. *The historical cost regime achieves an outcome close to the socially optimal solution when the credit risk is low (z_1 is close to 1), while it deviates from the socially optimal solution when the credit risk is high (z_1 is close to 0) by generating incentives for excessive holding.*

Proof. From (III.11), we see that as $z_1 \to 1$, the solutions approach the socially optimal values given by (III.6), while it departs more sharply from the socially optimal solution as $z_1 \to 0$. □

C. Fair value regime

An alternative to historical cost valuation is to measure assets at their fair value, i.e. their market price at the interim date. When the fair value of assets is used to specify the regulatory solvency constraint, risky assets are valued at the prevailing market value even if the insurer decides to hold on to the assets. Thus, the $t = 1$ FV solvency constraint for those insurers who choose to hold the long-term assets to maturity, given that a fraction s sells, is given by:

$$\frac{X}{v_1(s)} \leq c \tag{III.13}$$

We assume that the insurer faces a regulatory penalty proportional to its assets, $\rho * A_0(1 - b)$, if it decides not to sell the assets and breaches this regulatory constraint as prices fall. By contrast, insurers that decide to sell are not subject to a solvency constraint: they remain in operation if the sales proceeds exceed the amount due to policyholders at $t = 2$, while they are closed down otherwise.

For a given c, we define the threshold $\bar{s}^{FV}$, such that if the aggregate sales exceeds $\bar{s}^{FV}(c)$, then all the remaining insurers that do not sell will breach the solvency constraint (III.13):

$$\bar{s}^{FV} = \frac{c(z_1 R + \varepsilon) - X}{c\gamma}. \tag{III.14}$$

Therefore, under the FV regime, insurers will prefer to keep holding the risky asset, given a fraction s sells, only if its expected payoff accounting for the penalty from breaching the solvency constraint (III.14) is higher than the proceeds from selling:

$$\underbrace{z_1(R - X) - \rho * I(s > \bar{s}^{FV})}_{\text{Payoff when holding}} \geq \underbrace{[v_1(s) - X]^+}_{\text{Payoff when selling}} \tag{III.15}$$

where $I(s > \bar{s}^{FV})$ is an indicator function which takes the value one if $s > \bar{s}^{FV}$. Unlike the historical cost regime, the FV solvency requirement can become binding in the interim period if the fraction of sales is large enough. This is because in illiquid markets prices are sensitive to aggregate sales, which are therefore reflected in the insurers' solvency adequacy under fair value. As a result, insurers' behaviour under the fair value regime depends crucially on the specifications of the regulatory parameters, c and ρ. In particular, we define:

$$\varepsilon^{FV} \equiv \frac{X}{c} - z_1 R,$$

and

$$c^{FV} \equiv \frac{X}{z_1 R + (1 - z_1)X}. \tag{III.16}$$

Then the following proposition explains the equilibrium under the fair value regime.

Proposition III.4. ***(Fair Value Equilibrium)***

Under the fair value regime the equilibrium fraction of sales, denoted by s^{FV}, is:

- *the same as under the historical cost regime $s^{FV} = s^{HC}$, where s^{HC} is given by (III.11), when the FV constraint (III.13) does not bind for $s = s^{HC}$. Specifically, this is the case if $c > c^{FV}$ and $\varepsilon > \varepsilon^{FV}$. Under these assumptions, the resulting market price after the interim sales is given by (III.12).*
- *otherwise, if $\varepsilon < \varepsilon^{FV}$ or $c \leq c^{FV}$ the constraint (III.13) binds on the solution (III.11). Then in equilibrium*

  i) everyone sells $s^{FV} = 1$ when the penalty is large $\rho > z_1(R - X)$. In this case the equilibrium market price is $v_1(1) = max(z_1 R + \varepsilon - \gamma, 0)$.

  ii) while when the penalty is sufficiently low $\rho \leq z_1(R - X)$, the fraction of sales is given by:

$$s^{FV} = \begin{cases} 0 \text{ if } \varepsilon < X(1 - z_1) - \rho \\ \pi^{FV} \equiv \frac{1}{\gamma}[\varepsilon - X(1 - z_1) + \rho] \text{ if } X(1 - z_1) - \rho \leq \varepsilon \leq \gamma + X(1 - z_1) - \rho \\ 1 \text{ if } \varepsilon > \gamma + X(1 - z_1) - \rho \end{cases} \tag{III.17}$$

and the equilibrium prices are:

$$v_1^{FV} = \begin{cases} \max(z_1 R + \varepsilon, 0) \text{ if } \ \varepsilon < X(1 - z_1) - \rho \\ z_1 R + X(1 - z_1) - \rho \text{ if } X(1 - z_1) - \rho \leq \varepsilon \leq \gamma + X(1 - z_1) - \rho \\ z_1 R + \varepsilon - \gamma \text{ if } \varepsilon > \gamma + X(1 - z_1) - \rho \end{cases} \tag{III.18}$$

Proof See Annex.

Observe that FV generally leads to at least as much, or more sales than the HC regime, thus eliminating some of the excessive holding generated by HC. However, this can come at the cost of generating excessive sales when the regulatory parameters are not set appropriately. More specifically, we now examine how the equilibrium under the fair value regime compares with the socially optimal equilibrium depending on the regulatory parameters[11] c and ρ.

Proposition III.5. *Comparing the equilibrium under fair value to the socially optimal equilibrium, three possible cases can arise:*

1. *The fair value regime can achieve the socially optimal solution and the regulatory parameters are set appropriately. That is $c \leq c^{FV}$ and the penalty ρ for breaching the solvency constraint is set equal to*
$$\rho = X(1 - z_1). \tag{III.19}$$
2. *The fair value regime generates excessive asset sales by the insurers when $c \leq c^{FV}$ and $\rho > X(1 - z_1)$ for $X(1 - z_1) - \rho \leq \varepsilon \leq \gamma$.*

[11] Note that the equilibrium changes also relative to ϵ being lower or higher than ε^{FV}. However, since the market noise is not under the control of the regulator we ignore it in this discussion.

3. The fair value regime generates excessive holding when

i) $c \leq c^{FV}$ *and* $\rho < X(1 - z_1)$*, the equilibrium under FV represents excessive holding relative to the socially optimal solution in the parameter range* $0 < \varepsilon < \gamma + X(1 - z_1) - \rho$*. However, FV improves on the equilibrium under HC by reducing excessive holding.*

ii) $c > c^{FV}$*, the equilibrium under FV coincides with the one under HC, and represents excessive holding when* $\varepsilon > \varepsilon^{FV}$ *and* $0 \leq \varepsilon \leq \gamma + X(1 - z_1)$*.*

Proof See Annex.

Note that, unless the regulatory regime can be perfectly calibrated, there is a trade-off between two possible inefficient behaviours: risk-shifting through excessive holding, and fire sales due excessive selling. Specifically, when the penalty is relatively large, FV reduces excessive holding but at the cost of generating excessive sales. In this case, it is not clear in general whether FV outperforms or underperforms HC. This is because whereas fire sales under fair value are detrimental as they depress prices below the asset's fundamental value, possibly generating negative spirals, excessive holding under the historical cost regime leads to over-inflated market prices and exposes policyholders to losses in case of default. We expect HC to outperform FV when credit risk is low and the HC equilibrium is reasonably close to the socially optimal equilibrium. In contrast, FV generally outperforms HC as long as the penalty for breaching the solvency constraint under FV is set at moderate levels.

In principle, the fair value regime can generate the socially optimal behaviour if the regulator can set the regime (through ρ and c) optimally. However, there are practical challenges. First, the optimal penalty (III.19) corresponds to the expected amount due to policyholders in case of default (normalized by the assets value). This is equivalent to making insurers liable for the amount of losses they would not otherwise consider due to limited liability. Second, calibrating (III.19) optimally requires the regulator to be fully informed about the true state of the fundamental (z_1). However, it is sufficient for the regulator to be able to estimate a lower bound for credit risk (i.e. the lowest possible level of the default probability $1 - z_1$) in order to set the penalty appropriately (i.e. $\rho < X(1 - z_1)$), such that the fair value regime dominates historical cost. Since regulators collect very detailed, line-by-line asset data for insurers that are subject to the Solvency II regime, it is likely to be possible for them to make such estimates. However, the lack of flexibility in adjusting penalty miposed on those insurers that breach the solvency requirement could still lead to excessive punishment on those that breached the constraint due to market volatility despite having a fundamentally sound asset portfolios, thus prompting these insurers to sell in a falling market.

D. Policyholders' losses

So far we have compared the equilibria under the two valuation regimes to the one that is socially optimal, and in terms of asset sales and market prices. However, important real effects determined by insurers' behaviour are the losses for their policyholders. It is not clear a priori which regime performs better in this respect. While in general the risk-shifting behaviour generated under historical cost is detrimental for policyholders, when insurers are forced to sell at depressed prices they could impose losses on policyholders as well.

In particular, when insurers *hold* the asset, policyholders lose the entire insurance payout due at $t = 2$ if the asset defaults. Hence, the total expected amount of losses, as at $t = 1$, is:

$$L^{Hold} = (1 - z_1)D_2$$

Note when the asset does not default policyholders get paid the entire benefit and do not incur any losses, since by assumption (II.4) $R > X$. On the other hand, when a fraction s of insurers *sell* the asset, policyholders incur losses if the sales proceeds are not enough to cover the entire policy payout. In this case, the expected policyholders' losses are:

$$L^{Sell}(s) = \max\{D_2 - A_0(1 - b)(z_1 R + \varepsilon - \gamma\frac{s}{2}), 0\}$$

It is important to note that, unless all insurers are forced to sell, market prices remain high enough such that policyholders do not incur any losses. This is because in our model insurers sell sequentially and they can observe the market price. Hence, they would never sell at a loss, unless forced by regulatory solvency pressure. However, when the penalty is large enough to force everyone to sell, excessive sales can generate strictly positive losses for policyholders. Specifically, we have that

$$L^{Sell}(1) = D_2 - A_0(1 - b)(z_1 R + \varepsilon - \frac{\gamma}{2}) > 0,$$

when $\varepsilon < X - z_1 R + \frac{\gamma}{2}$, i.e. the market is relatively pessimistic and thus under-valuing the asset.

It follows that the expected losses at t=1 under HC are:

$$L_1^{HC} = \begin{cases} L^{Hold}, \text{if } \varepsilon < X(1 - z_1) \\ (1 - \pi^{HC})L^{Hold}, \text{if } X(1 - z_1) \leq \varepsilon \leq \gamma + X(1 - z_1) \\ L^{Sell}(1) = 0, \text{if } \varepsilon > \gamma + X(1 - z_1) \end{cases} \quad \text{(III.20)}$$

which coincides with the expected losses under FV when $c > c^{FV}$ and $\varepsilon > \varepsilon^{FV}$. By contrast, when $c < c^{FV}$ or $\varepsilon < \varepsilon^{FV}$ losses under FV are equal to:

$$L^{FV} = L^{Sell}(1) = \max(D_2 - A_0(1 - b)(z_1 R + \varepsilon - \frac{\gamma}{2}), 0)$$

if $\rho > z_1(R - X)$, and to

$$L^{FV} = \begin{cases} L^{Hold}, \text{if } \varepsilon < X(1 - z_1) - \rho \\ (1 - \pi^{FV})L^{Hold}, \text{if } X(1 - z_1) - \rho \leq \varepsilon \leq \gamma + X(1 - z_1) - \rho \\ L^{Sell}(1) = 0, \text{if } \varepsilon > \gamma + X(1 - z_1) - \rho \end{cases} \quad \text{(III.21)}$$

if $\rho < z_1(R - X)$. Then, comparing the expected losses when the two regimes differ, we have the following result.

Proposition III.6. *When $\rho < z_1(R - X)$, the expected policyholders losses under FV are less than, or equal to, losses under HC. The same holds when $\rho > z_1(R - X)$ and the market is optimistic, while FV generates larger expected losses than HC when the market is extremely pessimistic.*

Proof See Annex.

Therefore, we can confirm that, in general, HC leads to larger losses for policyholders than FV. This is because insurers excessively hold the risky assets which is detrimental for policyholders due to the possibility of default. On the other hand in general selling the asset is better for policyholders since it means raising cash to pay them back. However, when the penalty is large enough to make everyone sell and the market is pessimistic, the market price can become so depressed such that the losses from selling can exceed the ones from holding.

E. Market Adjusted Valuation

From the previous sections we can conclude that, in line with general views, both fair value and historical cost can generate inefficiencies. On the one hand, historical cost accounting enables insurers to hide the deterioration in the fundamental values of the assets they hold, and thus discourages them from selling the non-performing assets even when the market is offering relatively favorable prices. The reluctance of insurers to sell assets in turn leads to overinflated asset prices and larger policyholders losses. On the other hand, when insurers are subject to a solvency requirement which is based on fair value accounting, excessive asset sales and depressed asset prices are possible when the regulatory penalty for breaching the solvency constraint is set too severely.

Recognizing these problems associated with pure historical cost or fair value valuation regimes, policymakers have and are currently considering 'hybrid' approaches, which combine elements of the two regimes. In Europe, the Solvency II provisions on the Matching Adjustment and Volatility Adjustment are intended to mitigate the effect of short-term price movements on assets held longer term to meet liabilities to policyholders. For instance, [2] explain the role of the Matching Adjustment in the following way: "annuity writers often invest in long-term, illiquid bond assets that pay cash flows equivalent to the amount owed to policyholders. In this scenario, the annuity writers have matched their cash in-flows with their payments to policyholders and are only exposed to the risk of the bond issuer defaulting. The matching adjustment (MA) looks to address the balance sheet volatility that some insurers experience in the short term when using a market-consistent approach. It is a specific adjustment to the discount rate that insurers will be able to use to value certain predictable liabilities, for example, annuity payments. When a higher discount rate is used to calculate the present value of a firms expected future claims, the present value falls. By adjusting the liability side of the balance sheet, this retains the market-consistent valuation of assets, while reducing the impact of asset-price fluctuations on the balance sheet."

In the international setting, IAIS consulted on a Market Adjusted Valuation approach in 2016, as one of the valuation approaches explored in its development of an international Insurance Capital Standard (ICS)[12]. According the IAIS, ICS Principle 7 requires a valuation approach that prompts supervisory attention when appropriate. Such supervisory attention should not over-emphasise volatility that does not affect the solvency of an IAIG (Internationally Active Insurance Group). Prudentially sound behaviour by IAIGs is promoted where the ICS does not encourage IAIGs to take actions in a stress event that exacerbate the impact of that event (for example fire sales of assets) or to focus on short term goals to the detriment of appropriate long term objectives. Stability in valuation is important in that context[13].

[12] IAIS (2016) Risk-based Global Insurance Capital Standard ICS Consultation Document, `https://www.iaisweb.org/page/supervisory-material/insurance-capital-standard`

[13] Page 29 IAIS (2016) ibid.

Our setting is a fairly general application of adjustments to fair value, but does not exactly replicate the features of any specific regime, current or in development. Hence, in the following analysis, we refer to Market Adjusted Valuation as a general adjustment, rather than being specific to any existing regulator regime. Broadly speaking, the market adjusted valuation (MAV) aims to reduce the impact of 'noise' in market prices, i.e. price movements that are not reflective of changes in default risk of the asset, on regulatory solvency constraints by adjusting the discount rate on insurers' liabilities appropriately. In fact, changes in asset prices not attributed to credit risk should not impact insurers' solvency, so long as the insurer does not need to sell the asset. Discounting liabilities' at a higher rate when asset values fall, causing liabilities to fall similarly, would alleviate (at least in part) solvency pressure.

Hence, the key question for MAV is how to design the discount factor on insurers' liabilities, in order to adjust the fluctuations in market prices which affect the asset side of insurers' balance sheets. As we have observed previously, the FV regime can achieve the optimal outcome as long as the regulator can distinguish the fluctuations in fundamental credit risk from noise, and set the penalty for breaching the solvency constraint appropriately. Thus, the relevant question is whether a MAV-based solvency regime dominates a FV-based solvency regime when the regulator is not fully informed about whether the fluctuations in asset prices reflect noise or changes in fundamentals. This section examines how such a regime might perform relative to FV and HC regimes.

One approach is for MAV to discount liabilities at a rate which is proportional to the asset's total spread. The proportion, which we denote by $\theta \in [0,1]$ is a constant set by the regulator. The deducted proportion of the spread should capture the asset credit risk, and the remaining proportion the price movements that are not reflective of changes in default risk. In this way the discount factor would alleviate solvency pressures due to market volatility and noise while keeping the pressure due to an increase in credit risk. Such an approach would better reflect the long-term nature of insurance liabilities, that allows the insurers to earn part of that spread risk-free.

In our stylized framework, we define the liabilities discount rate as:

$$\delta \equiv 1 + \theta\alpha \tag{III.22}$$

that is the risk-free rate, normalized to one, plus a proportion of the total asset spread[14] α:

$$v_1(s) = \frac{R}{1+\alpha} \Leftrightarrow \alpha \equiv \frac{R - v_1(s)}{v_1(s)}.$$

It follows that the solvency constraint under MAV can be written as:

$$\frac{X}{\delta v_1(s)} \leq c. \tag{III.23}$$

As we would expect, since $\delta \geq 1$, the MAV constraint (III.23) relaxes the FV constraint (III.13) for any given c, except in the special case when there is no discount of liabilities, i.e. $\delta = 1$ and the MAV constraint coincides with the FV constraint. Substituting (III.22) in (III.23), we can rewrite the MAV solvency constraint as follows:

$$\frac{X}{\theta R + (1-\theta)v_1(s)} \leq c \tag{III.24}$$

[14]Note that since $\varepsilon \leq (1-z_1)R$, we have that $v_1(0) \leq R$, hence the spread is always positive.

Thus, under MAV, the regulator is basically evaluating assets as a weighted average of their market price $v_1(s)$ and their final return in the absence of default R, where the weight on the final return is given by θ. In what follows, we do not make any restrictions on the value of θ, but we will show with some numerical examples how the regime performs for different choices of this parameter. Define the threshold $\bar{s}^{MAV}$ such that, for any given c, if the aggregate sales exceeds $\bar{s}^{MAV}$, then all the remaining insurers that do not sell will breach the solvency constraint (III.24):

$$\bar{s}^{MAV} = \frac{(1-\theta)(z_1 R + \varepsilon) + \theta R - \frac{X}{c}}{\gamma(1-\theta)}. \tag{III.25}$$

Note that $\bar{s}^{MAV}(c) < 0$ if $\varepsilon < \varepsilon^{MAV} \equiv \frac{1}{(1-\theta)}\left[\frac{X}{c} - (1-\theta)z_1 R - \theta R\right]$. Under this condition, when the market is sufficiently pessimistic, the MAV constraint binds for all insurers even when no one sells. It follows that, as under the FV regime, under MAV insurers will prefer holding the assets, conditional on a fraction s selling, if:

$$\underbrace{z_1(R-X) - \rho * I(s > \bar{s}^{MAV})}_{\text{Payoff when holding}} \geq \underbrace{[v_1(s) - X]^+}_{\text{Payoff when selling}} .$$

Define:

$$c^{MAV} \equiv \frac{X}{\theta R + (1-\theta)\left[z_1 R + (1-z_1)X\right]}$$

then the equilibrium under MAV can be described as follows.

Proposition III.7. ***(Market Adjusted Valuation Equilibrium)***
Under market adjusted valuation the equilibrium fraction of sales, denoted by s^{MAV}, is:

- *the same as under the historical cost regime $s^{MAV} = s^{HC}$, where s^{HC} is given by (III.11), when the MAV constraint (III.23) does not bind for $s = s^{HC}$. This occurs if $c > c^{MAV}$ and $\varepsilon > \varepsilon^{MAV}$.*
- *otherwise, when $\varepsilon < \varepsilon^{MAV}$ or $c < c^{MAV}$ the constraint (III.23) binds on the HC solution. Then the equilibrium solution is the same as under the fair value regime:*

  i) when $\rho > z_1(R-X)$ everyone sells $s^{MAV} = 1$ and the market price is $v_1^{MAV} = \max(z_1 R + \varepsilon - \gamma, 0)$.

  ii) when $\rho \leq z_1(R-X)$, the equilibrium fractions of sales and market prices are given respectively by (III.17) and (III.18).

Proof See Annex.

Note that analogously to the FV regime, MAV can replicate the social planner's solution by setting the regulatory parameters appropriately: $c \leq c^{MAV}$ and $\rho = X(1-z_1)$. Apart from this particular case, when the regulatory parameters are set too loose or too tight, MAV generates inefficient behaviours as under HC or FV respectively. Specifically, three different cases can arise.

First, the *three regimes generates the same equilibrium* when $c > c^{FV}$ and $\varepsilon > \varepsilon^{FV}$[15]. This is because the FV and MAV constraints do not bind in this case, and thus all the regimes generate excessive holding (for $0 < \varepsilon < \gamma + X(1-z_1)$).

[15] Note that $c^{FV} > c^{MAV}$ and $\varepsilon^{FV} > \varepsilon^{MAV}$ (this is true because $c > \frac{X}{R}$ given that insurers satisfy $t = 0$ solvency constraint (II.2)). Therefore $c > c^{FV}$ and $\varepsilon > \varepsilon^{FV}$ imply that $c > c^{MAV}$ and $\varepsilon > \varepsilon^{MAV}$.

< Figure 3 about here >

Second, *MAV coincides with HC but deviates from the FV equilibrium* when the solvency constraint is set loose enough, i.e. $c^{MAV} < c < c^{FV}$ and $\varepsilon^{MAV} < \varepsilon < \varepsilon^{FV}$. Then MAV generates excessive holding (for $0 < \varepsilon < \gamma + X(1 - z_1)$) and is dominated by FV dominates as long as the penalty is set low enough $\rho \leq X(1 - z_1)$, as FV reduces excessive holding.

< Figure 6 about here >

However, if the penalty from breaching the requirement ρ is set too high ($\rho > X(1 - z_1)$), it is not clear whether MAV (and HC) outperforms or underperforms FV: whereas FV generates excessive sales, HC generates excessive holding. MAV eliminates the problem of excessive sales generated by FV but only by re-introducing the problem of excessive holding under HC. Proposition III.3 implies that MAV (and HC) would be expected to outperform FV if credit risk is relatively low, and there is limited incentives for insurers to risk-shift to policyholders.

Finally, *MAV and FV coincide, but both deviate from the HC equilibrium* if the solvency ratio constraints are set relatively tightly, i.e. $c < c^{FV}$ (or $\varepsilon < \varepsilon^{FV}$), and $c < c^{MAV}$ (or $\varepsilon < \varepsilon^{MAV}$). In this case MAV and FV outperform HC if the penalty is not too high ($\rho < X(1 - z_1)$) since they reduce excessive holding[16]. By contrast, both regimes generate excessive sales if $\rho > X(1 - z_1)$, in which case no regime strictly dominates the others, as shown in the figure below.

< Figure 9 about here >

In conclusion, MAV simply switches between FV and HC depending on whether the solvency constraint becomes binding, and hence does not simultaneously outperform both. When the solvency constraint under MAV binds, the regime generates excessive sales exactly as FV. This underscores the point that a hybrid regime, such as MAV, does not necessarily prevent procyclical behaviour and fire sales. Instead, the solution can be provided within the FV regime, as long as the regulators can adjust the penalty depending on the fundamental credit risk in insurers' asset portfolio (z_1 in the model). Our model suggests that the regulator should lower the penalty when they judge that the FV solvency constraint was breached even though the insurers' asset portfolios are fundamentally sound. Thus, we conclude that the regulators' ability to control the tightness of the regime by varying the penalty imposed on those insurers that breach the constraint according to the fundamental quality of their balance sheets is at least as important, or possibly more important, than the valuation regime itself.

F. Numerical examples

We now present some numerical examples, to illustrate the main results of the model. In particular, we provide a comparison of the three different valuation regimes with the socially optimal solution, in terms of the real effects they generate on asset prices and policyholders losses.

In the baseline simulation, we set the key parameters as follows: $R = 1.01$, $z_1 = 0.9$, $\theta = 0.5$, $D_1 = 0.50$, $D_2 = 0.49$, and $\rho = 0.01$. The calibration of the underwriting parameter is based on mortality rates of 1% over one year for a man aged 60, using data from the UK's Office for National Statistics. Assuming that the ex-ante probability of downgrade is $p = 1\%$, the ex-ante default probability on assets held by insurers

[16] MAV and FV still generate excessive holding but only for $0 < \varepsilon < \gamma + X(1 - z_1) - \rho$.

is given by $p * (1 - z_1) = 0.1\%$. This value is consistent with the estimates of compensation investors require for expected and unexpected defaults that are obtained using data from the decomposition of sterling investment-grade corporate bonds[17]. Capital charges for bonds are not straightforward to derive, as they depend on factors such as credit quality and duration, contributing to spread risk, and changes in interest rates (although this is not present in our model). Informed by the data for several firms where more detailed information is available on the breakdown of total capital requirements by type of risk, we set our mark-down of the value of bonds c as approximately 80% in the baseline analyis, but we also explore the implications of changing this parameter. The market depth γ has been obtained using liquidity compensation data from JP Morgan DataQuery. Specifically, we have decomposed the residuals from the sterling investment-grade corporate bond spreads after accounting for credit risk compensation, into compensation for liquidity and an unexplained, or a "noise" component. Informed by these data we set $\gamma = 5\%$. implies that, when all insurers sell an asset, the price of that asset will fall by 5.5% relative to its fundamental value ($\frac{\gamma}{z_1 R} = 0.055$).

Further, we assume that insurers start $t = 0$ with capital to investable asset ratio (E_0/A_0) of 35%: thus, we set $E_0 = 0.54$ given that the insurers collect one unit of premium, implying that $A_0 = E_0 + 1 = 1.54$. This choice was informed by Solvency II regulatory returns gathered by the Prudential Regulation Authority in 2016, which provided information about life insurers' balance sheets and capital requirements. The median value of the ratio of assets to liabilities for large companies that specialise in underwriting products with guarantees, consistent with our calibration, is approximately 1.35. Note that given these values, the penalty ρ is lower than $X(1 - z_1) = 0.047$, for the baseline calibration.

Under this calibration, the three regimes coincide and generate too little sales relative to the socially optimal sales at $t = 1$ when the market is moderately optimistic (i.e. ε is moderately positive), see Figure 3. As a result, as shown in Figure 4, when the market is moderately optimistic, the asset price is overinflated relative to the fundamental value. Note that, even under the socially optimal decision rule, over- and under-valuation of assets can occur in the presence of market noise. The expected policyholder losses generated under the three regimes are higher than the social optimum, as insurers are reluctant to sell downgraded assets, even when prices are favourable (see Figure 5).

Figure 6 and 7 show the equilibrium sales and the resulting market prices under an alternative scenario, in which the asset mark-down c is lowered to 0.49, such that $c^{MAV} < c < c^{FV}$, while maintaining the baseline calibrations for other parameters. A lower c could be justified if the regulator expects larger stressed losses and thus requires insurers to have more capital. In this case, FV outperforms MAV which coincides with HC, by encouraging more sales when the market price is favourable and thus reducing the inefficient holding of downgraded assets. As a result, policyholder losses are lower under FV than under MAV and HC, as shown in Figure 8.

Finally, we consider the case when MAV and FV coincide, and both generate excessive sales: to do this, we set $c = 0.47 < c^{MAV}$ and $\rho = 0.1 > X(1 - z_1)$, while maintaining baseline calibrations for other parameters. Figures 9 and 10 show that, under both FV and MAV, insurers dumping into the market lower prices when the market is already depressed. This occurs because the penalty for breaching the solvency constraint is set at a high level. Interestingly, policyholder losses (see Figure 11) are lower than under the socially optimal case, implying that the cost of inefficient sales are borne entirely by insurers' shareholders as long as the insurers remain solvent. This example highlights the potentially conflicting objectives that a regulator can face between protecting policyholders and preventing asset price volatility

[17] These estimates are regularly produced by the Bank of England, see for instance [3]. The methodology is based on [14].

which could destablise the financial system, that is between micro- and macroprudential regulation.

IV. Ex-ante asset choice

Thus far, we have taken insurers' asset portfolio as given[18], in order to focus our analysis on the impact of solvency requirements on *ex-post* insurers' investment behaviour. Starting from the same asset allocation allowed us to obtain a cleaner comparison between the different valuation regimes. However, valuation rules, through their interaction with solvency requirements, can generate important incentives ex-ante when insurers decide how to allocate their fundings among different asset classes. In particular, forward-looking insurers who anticipate future solvency pressures might adopt more prudent investment strategies, relative to those who are less constrained by regulation.

Thus, in this section, we consider how expectations about future downgrades and associated solvency pressures, influence insurers' *ex-ante* asset allocation when different valuation regimes are in place. As before at $t = 0$ insurers must hold enough safe assets to meet the $t = 1$ insurance payout. This implies that the minimum fraction needed to be invested in safe assets is

$$b \geq \hat{b} \equiv \frac{D_1}{A_0}. \tag{IV.1}$$

We now examine how the insurers will allocate the remaining assets by optimizing between the risk-free and the risky asset. More specifically, at $t = 0$, insurers have a choice of investing the funds remaining after meeting the requirement (IV.1) either in the safe asset, which yields a return normalised to 1, or in the risky asset which is subject to a possibility of a credit downgrade at $t = 1$. As at $t = 0$, the probability of a downgrade is p, so the expected return of the risky asset is $[(1-p)+pz_1]R > 1$. To ensure tractability we only allow for two different strategies: either insurers invest everything in the safe asset, i.e. $b = 1$, or only the minimum to meet the constraint (IV.1), i.e. $b = \hat{b}$. In the latter case the asset allocation remains (II.1) as considered in the previous sections. We thus perform a comparison between the safest and the riskiest portfolios possible, which allow us to obtain clear predictions on the asset allocation chosen under different valuation regimes. As we will show, insurers will choose the riskiest portfolios, for different values of the probabibility of downgrade p under different valuation regimes.

Note that, if insurers invest all their funds in the safe asset, they will never breach the solvency constraint at $t = 1$, provided that their initial equity is large enough to satisfy the solvency constraint at $t = 0$, as specified in (II.3)[19]. By contrast, if they invest in the risky asset, they need to consider the possibility of breaching the constraint in the interim period due to a fall in asset prices, if their assets are measured by their market values. Thus, insurers will compare the expected return from the safe asset with the one from the risky asset, taking into account the expected market value in the interim period and the possibility of breachign the solvency constraint. In particular, at $t = 0$ insurers do not know their

[18] Specifically we have assumed that the fraction invested in the risk-free asset is (II.1) and the remaining $1 - b$ is invested in the risky asset. Note that this asset allocation is the optimal asset allocation of an insurers who cannot anticipate at t=0 the asset downgrade at t=1.

[19] Note that when insurers invest all their funds in the risk-free asset, the initial constraint should be

$$E_0 \geq D_1 + D_2 - 1.$$

This is a looser requirement relative to (II.3), implying that less initial equity is required to set up the company. In the case the insurance contract is priced at fair value, i.e. $D_1 + D_2 = 1$, this constraint is never binding. However, we do not take into account the cost of equity in the asset allocation problem and just take the initial equity as given.

place in the queue of sellers, therefore they have to base their decision on expected prices:

$$v_0(s) = E_0[v_1(s)] = E_0[z_1 R + \varepsilon - \gamma s] = z_1 R + E_0[\varepsilon] - \gamma \frac{s}{2},$$

where $v_1(s)$ is the market price at $t = 1$. The above expression uses the fact that, at $t = 0$, the place in the queue of a given insurer that sell at $t = 1$ is uniformly distributed in $[0, s]$, if s is the total fraction of insurers that sell at $t = 1$. In the following, we restrict the parameter ε^H to be larger than $\gamma + X(1 - z_1)$, in order to cover all possible cases at $t = 1$.

First, we characterize the asset allocation that maximizes social welfare, given by insurers' and policyholders' expected value. Under the safe strategy $b = 1$, the social welfare is simply equal to the insurers' total initial assets value A_0, while under the risky strategy $b = \hat{b}$ the social welfare depends on the probability p of a downgrade. In particular, it is given by

$$A_0 \hat{b} + (1 - p) A_0 (1 - \hat{b}) R + p A_0 (1 - \hat{b}) w^s$$

where w^s is the expected social welfare given a downgrade and given that the socially optimal rule (III.6) is followed at $t = 1$:

$$\begin{aligned} w^s \quad = \quad & \int_{\varepsilon_L}^{0} z_1 R f(\varepsilon)\, d\varepsilon + \int_0^{\gamma} \left[\frac{\epsilon}{\gamma} \left(z_1 R + \varepsilon - \frac{\gamma}{2} \frac{\varepsilon}{\gamma} \right) + (1 - \frac{\varepsilon}{\gamma}) z_1 R \right] f(\varepsilon)\, d\varepsilon \\ & + \int_{\gamma}^{\varepsilon_H} \left[z_1 R + \varepsilon - \frac{\gamma}{2} \right] f(\varepsilon)\, d\varepsilon \end{aligned}$$

Thus, it is socially optimal to invest in risky asset as long as

$$\hat{b} + (1 - p)(1 - \hat{b}) R + p(1 - \hat{b}) w^s > 1 \tag{IV.2}$$

Proposition IV.1. ***(Socially optimal ex-ante asset allocation)***
The risky strategy $\hat{b}$ is socially optimal if and only if $R + p(w^s - R) > 1$. This implies that:

- *If $w^s > R$, the risky strategy $\hat{b}$ is always socially optimal, for any probability of downgrade, p.*
- *If $w^s < R$, the risky strategy $\hat{b}$ is socially optimal only if*

$$p < p^* \equiv \frac{R - 1}{R - w^s}.$$

Proof *The decision rule (IV.2) simplifies to $R + p(w^s - R) > 1$, which can be solved for p to yield the above results.*

Note the socially optimal asset allocation differs from the one minimizing policyholders' losses, which would make insurers invest everything in the risk-free asset for any value of p. This is because by doing so policyholders would not incur any losses, while by investing a fraction $\hat{b}$ in the risky asset they would incur losses if the asset defaults.

Let us now analyze the insurers' asset allocation under different valuation regimes. If insurers choose the safe investment strategy $b = 1$, their expected payoff is independent of the valuation regime, and given by their initial assets net of the payments to policyholders:

$$A_0 - D_1 - D_2 = A_0(1-\hat{b})(1-X).$$

For ease of comparison with the risky strategy $\hat{b}$, we have expressed this payoff in terms of the interim liabilities-to-assets ratio under the risky strategy $X = \frac{D_2}{A_0(1-\hat{b})}$. When insurers choose the risky strategy they become exposed to the possibility of a downgrade at $t = 1$. In this case, the valuation rule used to define the solvency requirement after the downgrade plays a key role.

In particular, *under the historical cost regime*, when the risky asset is downgraded, which happens with probability p, insurers will only sell the asset if its market price is high enough, as described by (III.11). By contrast, with probability $1-p$ the asset will not be downgraded and all insurers will hold it to maturity obtaining a certain return R. Thus, the expected payoff from the risky strategy under HC is given by:

$$(1-p)(RA_0(1-\hat{b}) - D_2) + pA_0(1-\hat{b})w^{HC} = A_0(1-\hat{b})[(1-p)(R-X) + pw^{HC}]$$

where w^{HC} is the insurers expected payoff in the case of a downgrade, given that they will follow the optimal rule (III.11) at t=1:

$$w^{HC} = \int_{\varepsilon_L}^{(1-z_1)X} z_1(R-X)f(\varepsilon)\,d\varepsilon$$
$$+\int_{(1-z_1)X}^{\gamma+X(1-z_1)} \left\{\pi^{HC}\left[z_1R+\varepsilon-\gamma\left(\frac{\pi^{HC}}{2}\right)-X\right]+(1-\pi^{HC})z_1(R-X)\right\} f(\varepsilon)\,d\varepsilon$$
$$+\int_{\gamma+X(1-z_1)}^{\varepsilon^H}[z_1R+\varepsilon-\tfrac{\gamma}{2}-X]f(\varepsilon)\,d\varepsilon$$

Proposition IV.2. ***(Historical cost regime ex-ante asset allocation)***
Under the historical cost regime insurers will choose the risky strategy $\hat{b}$ if

$$(R-X) + p(w^{HC} - (R-X)) > 1 - X. \tag{IV.3}$$

This implies that

- *If $w^{HC} > R - X$ insurers will always choose the risky strategy, for any value of p.*
- *If $w^{HC} < R - X$ insurers will choose the risky strategy only if*

$$p < p^{HC} = \frac{R-1}{R-X-w^{HC}}.$$

Proof *The above follows straightforwardly by solving (IV.3) for p.*

By contrast, *under fair value*, insurers' decision will also be driven by expectations of future solvency pressures, and thus will be highly dependent on the regulatory parameters. In order to focus our attention on the interesting case where FV and HC differ, in the following we assume that $c \leq c^{FV}$ and analyze the *ex-ante* asset allocation under the fair value regime for different values of the penalty ρ.

In particular, at $t = 0$ the expected payoff for insurers given a downgrade is given by:

$$
\begin{aligned}
w^{FV} &= \int_{\varepsilon_L}^{(1-z_1)X-\rho} z_1(R-X)f(\varepsilon)\,d\varepsilon \\
&+ \int_{(1-z_1)X-\rho}^{\gamma+(1-z_1)X-\rho} \left\{\pi^{FV}\left[z_1R+\varepsilon-\gamma\left(\frac{\pi^{FV}}{2}\right)-X\right]+(1-\pi^{FV})\left[z_1(R-X)-\rho\right]\right\} f(\varepsilon)\,d\varepsilon \\
&+ \int_{\gamma+(1-z_1)X-\rho}^{\varepsilon^H} \left[z_1R+\varepsilon-\frac{\gamma}{2}-X\right] f(\varepsilon)\,d\varepsilon
\end{aligned}
$$

if the penalty is small enough $\rho \leq z_1(R-X)$, such that they will follow the optimal rule (III.17) at $t = 1$, and by:

$$
w^{FV} = z_1(R-X) + \int_{\varepsilon^L}^{\varepsilon^H} (\varepsilon - \frac{\gamma}{2} - X(1-z_1))f(\varepsilon)\,d\varepsilon
$$

when the penalty is large $\rho > z_1(R-X)$ and everyone sells at $t = 1$ in equilibrium. Then, under the fair value regime, insurers' expected payoff under the risky strategy $b = \hat{b}$ can be written as:

$$
(1-p)(RA_0(1-\hat{b})-D_2) + pA_0(1-\hat{b})w^{FV} = A_0(1-\hat{b})[(1-p)(R-X)+pw^{FV}]
$$

Proposition IV.3. ***(Fair value regime ex-ante asset allocation)***
Under the fair value regime insurers will choose the risky strategy $\hat{b}$ if and only if:

$$
(R-X) + p(w^{FV} - (R-X)) > 1 - X. \tag{IV.4}
$$

This implies that

- *If $w^{FV} > R - X$ insurers will always choose the risky strategy for any value of p.*
- *If $w^{FV} < R - X$ insurers will choose the risky strategy only if*

$$
p < p^{FV} = \frac{R-1}{R-X-w^{FV}}.
$$

Characterization IV.4. Proof *The above follows straightforwardly by solving (IV.4) for p.*

We now want to compare the asset allocations under HC and FV and to the socially optimal asset allocation. Note that while the safe strategy is always optimal from the policyholders prospective, the risky strategy generates higher returns for shareholders. Thus, if credit risk is not elevated, the risky strategy can be socially optimal. However, when insurers are not restricted by regulation, since they maximize shareholders' value and are protected by limited liability, they have incentives to undertake excessive risks. In particular, they will prefer to invest in the risky strategy even for higher levels of credit risk than socially optimal, at the expense of policyholders. This is exactly what happens under the historical cost regime. Insurers anticipate that there will be no solvency pressure due to future credit downgrades, since assets are measured at their book value, and will therefore undertake the risky strategy for higher levels of p. Hence, we can conclude that historical cost leads to excessive risks both *ex post*, by 'gambling for resurrection' through excessive holding of downgraded assets, and *ex ante* by inducing riskier asset allocations.

By contrast, under the fair value regime, insurers' regulatory solvency will depend on the asset credit risk. Thus insurers, anticipating the possibility of a downgrade and the associated solvency pressures, will undertake a more prudent asset allocation than under the historical cost regime. In the extreme, when insurers are subject to a stringent regulation, with severe punishments from breaching the solvency constraint, their asset allocation will be even more prudent than what would be socially optimal. These results are established formally in the next proposition.

Proposition IV.5. *The following properties hold:*

i) $w^{HC} > w^{FV}$

ii) $w^{HC} + X > w^s$

iii) $w^s > w^{FV} + X$ *when* $\rho > z_1(R - X)$ *and when* $X(1 - z_1) \leq \rho \leq z_1(R - X)$ *if:* ***[To discuss: do we need*** $z_1 R > X$***, or not? Can we just state 'when*** $\rho > X(1 - z_1)'$?]

$$\rho(\rho + \gamma) > X\,(1 - z_1)\,[X(1 - z_1) + \gamma - 2\varepsilon^L] \tag{IV.5}$$

But $w^s < w^{FV} + X$ *when* $\rho < X(1 - z_1)$.

Therefore, when $w^s > w^{FV} + X$*, we have that:*

$$p^{FV} < p^* < p^{HC}.$$

Proof See Annex.

Thus, under FV insurers would invest more prudently than under HC, and also more prudently than the socially optimal allocation, while under HC insurers would invest less prudently than what socially optimal. By contrast, when the penalty is sufficiently low, $\rho < X(1 - z_1)$, FV leads to a riskier asset allocation than socially optimal but still safer than under historical cost. This is in fact the case when FV leads to excessive holding of then risky asset at $t = 1$. Finally, in the special case $\rho = X(1 - z_1)$, FV leads to the socially optimal solution If $\varepsilon^L = 0$ then $w^s = w^{FV} + X$ and FV leads also to the socially optimal *ex-ante* asset allocation. Therefore, the *ex-ante* analysis confirms the importance of considering the regulatory framework rather than the valuation regime in isolation.

Our results are in line with empirical evidence on investment behaviour of the insurance industry. For instance, the Procyclicality Working Group (2014) finds some evidence that insurers and pension funds subject to prudential regulation based on market values have moved away from equities toward long-duration bonds. Consistent with our findings, [7] have found that while US insurers subject to mark-to-market accounting were more likely to sell downgraded assets being hit by large price declines, they also adopted more prudent portfolio strategies relative to insurers that were subject to historical cost accounting. Specifically, they find that their portfolio allocation is safer both across asset classes and for the choice of securities in each class. Our model is able to generate both predictions: in our framework, insurers under the fair value regime are more likely to invest everything in the safe asset, and when they invest in risky assets they choose securities with a lower probability of downgrade, relative to those under the historical cost regime.

To the best of our knowledge, we are the first to rationalize this effect analytically. While a significant amount of work has been done to analyze the effects of accounting rules during crisis times, less attention has been paid on their effects on portfolio choices in normal times before shocks materialize. Our model shows that while fair value can generate fire sales and amplify shocks during downturns, it can also lead to a more prudent asset allocation before the build up of the crisis. By contrast, historical cost dampens selling pressures during periods of price declines, but at the cost of inducing excessive risk taking both in normal and in crisis times.

V. Ex-ante policyholders' losses

We conclude the paper by analyzing the expected policyholders' losses at $t = 0$, accounting for the *ex-ante* asset allocation. Note that when insurers choose the safe strategy, investing everything in the risk-free asset, policyholders do not suffer any losses. Therefore, we only need to compare losses, under the historical and fair value regimes, when insurers undertake the risky strategy ($b = \hat{b}$) and there is a downgrade. This is because if there is no downgrade, the asset does not default and insurers pay pack all policyholders at $t = 2$.

Define the expected policyholders losses under HC as:

$$L_0^{HC} = pE_0[L_1^{HC}]$$

where $E_0[L_1^{HC}]$ is the $t = 0$ expectation of the expected policyholders losses L_1^{HC} at $t = 1$, given by (III.20), and the expected losses under FV as:

$$L_0^{FV} = pE_0[L_1^{FV}]$$

where L_1^{FV} denote the expected policyholders losses at $t = 1$. In particular, this is equal to (III.20), as under HC, when $c > c^{FV}$ and $\varepsilon > \varepsilon^{FV}$. By contrast, when $c < c^{FV}$ or $\varepsilon < \varepsilon^{FV}$, if the penalty is large $\rho > z_1(R - X)$ then in equilibrium everyone sells and

$$L^{FV} = L^{Sell}(1),$$

while if the penalty is sufficiently small $\rho < z_1(R - X)$, the expected losses are given by (III.21). Finally, we denote the expected policyholders losses under the socially optimal strategy (III.6) as:

$$L_0^* = pE_0[L_1^*],$$

where

$$L_1^* = \begin{cases} L^{Hold}, \text{if } \varepsilon < 0 \\ (1 - \pi^*)L^{Hold}, \text{if } 0 \leq \varepsilon \leq \gamma \\ L^{Sell}(1) = 0, \text{if } \varepsilon > \gamma \end{cases} \tag{V.1}$$

It is important to note that, as we have shown in the previous section V, the probability p of a downgrade of the chosen risky asset can differ from one regime to the other. In particular, under HC p is always higher or equal to the one under FV. Then, comparing the losses for policyholders when the historical cost and fair value regimes differ, for $c < c^{FV}$, we obtain the following results.

Proposition V.1. ***(Policyholders' Losses)***

The expected losses for policyholders under HC are always higher than those under the socially optimal asset allocation. By contrast, the expected losses under FV depends on the penalty ρ*:*

- *When* $\rho \leq X(1 - z_1)$*, expected losses under FV are lower than under HC, but larger than the losses under the socially optimal asset allocation.*
- *When* $X(1 - z_1) \leq \rho \leq z_1(R - X)$*, policyholders losses under FV are lower than under HC, and also lower than the losses under the socially optimal asset allocation when (IV.5) holds.*
- *When* $\rho > z_1(R - X)$*, FV generates lower losses than HC at* $t = 1$ *if the market is relatively optimistic, while it leads to higher losses than HC when the market is pessimistic. However, because the ex-ante asset allocation is more prudent, the overall effect on policyholders is ambiguous. Relative to the socially optimal asset allocation, the expected losses are larger, but again because the ex-ante asset allocation is safer the overall effect on policyholders is ambiguous.*

Proof See Annex.

To sum up, measuring assets at historical cost for prudential regulation encourages insurers to behave in a way which increases the risk exposures of policyholders. This is because insurers would invest in riskier assets and would prefer holding on to them even after a downgrade, unless market conditions are particularly favourable. Measuring assets at their market value instead can reduce losses for policyholders when the regulatory penalty for breaching the FV solvency requirement is not too severe. In particular, when the penalty is very low, losses under fair value are even lower than those under the welfare maximizing behaviour. Thus, we can conclude that, in general, FV performs better than the HC as long as the penalty for breaching the solvency constraint is not too severe.

By contrast, if the penalty is set too high, insurers' procyclical behaviour during downturns can be detrimental for policyholders. This is because insurers become forced sellers when market prices are particularly unfavourable, forcing them to realize losses. But even in this case, it does not imply that policyholders under the FV regime are worse off than under the HC regime. In fact, in order to avoid the situation just described, under fair value insurers would choose assets with a lower probability of being downgraded, thus reducing the chances of exposing policyholders to losses. Thus, when the regulatory penalty for breaching the FV constraint is set too high, the relative welfare implications of the HC regime versus the FV regime are ambiguous.

VI. Conclusions

What is the appropriate valuation regime for solvency regulation for financial institutions with long-term liabilities, held by 'uninformed' policyholders, such as life insurers? Our analysis suggests that, for such institutions, the historical cost regime is undesirable because it could encourage insurers to hide the deterioration in asset quality and thus ultimately magnify losses for policyholders. The ability to hide losses *ex post* makes them also more likely to also encourage them to hold riskier portfolios *ex ante*. By contrast, the fair value regime could encourage fire sales following a negative shock to asset prices if the regulatory penalty for breaching the solvency constraint is too severe. In general, we expect the presence of market discipline to also encourage insurers to invest in less risky assets *ex ante*. Our analytical results are consistent with [7] empirical findings that US insurers that were subject to historical cost accounting were less likely to sell in a falling market, but more likely to hold riskier portfolios, relative to those insurers that were subject to the fair value accounting regime.

We have shown that, if the regulator can observe the quality of insurers' asset portfolios, then it can achieve the socially optimal outcome by adjusting the regulatory penalty associated with breaching the fair value solvency ratio according to the quality of insurers' investment portfolio. In particular, we show that the regulators should be lenient towards those insurers, which have fundamentally sound asset portfolios but have breached the solvency constraint due to a fall in asset prices. A pre-commitment to such a policy leads to both efficient portfolio *ex ante*, and efficient asset sales *ex post*. Thus, such a policy would be time-consistent. More generally, our results suggest that, a fair value solvency outperforms historical cost regime as long as the regulatory penalty for breaching the fair value solvency constraint is not too stringent.

Our results therefore suggest that a solvency regime based on market valuation could be made effective as long as regulators have good information about insurers' balance sheets, and there is scope for regulators to reduce or suspend penalties imposed on insurers that breach the solvency constraint due to severely stressed financial market conditions. Our analysis also suggests that a hybrid valuation regime, which does not fully reflect changes in market values for reasons other than deterioration in quality, can outperform fair value when the penalty is set too high, and the credit risk is relatively low. However, by itself it does not improve on the fair value regime without reintroducing the problems associated with the historical cost regime. As in the case of the fair value regime, such a hybrid regime would deliver a better outcome if the regulator were able to treat those insurers that breached the solvency constraint *despite* good asset quality, and those that did so *because of* bad asset quality, differently. Thus, finding a 'Goldilocks' valuation regime may not on its own be capable of addressing the dual problems of asset fire sales and gambling for resurrection. A more promising approach might be to allow regulators to suspend temporarily actions against solvency breaches which are simply reflective of market dislocation and not credit quality, when asset prices influence regulatory solvency ratios. To enable this, supervisors will need to be confident about asset valuation to inform the appropriate regulatory response.

References

[1] Franklin Allen and Elena Carletti. Mark-to-market accounting and liquidity pricing. *Journal of accounting and economics*, 45(2):358–378, 2008.

[2] Bank of England. The prudential regulation of insurers under Solvency II, Bank of England Quarterly Bulletin 2015. 2015.

[3] Bank of England. Bank of England Inflation Report August 2016. 2016.

[4] Bank of England and the Procyclicality Working Group. *Procyclicality and structural trends in investment allocation by insurance companies and pension funds*. Bank of England, 2014.

[5] Katrin Burkhardt and Roland Strausz. Accounting transparency and the asset substitution problem. *The Accounting Review*, 84(3):689–712, 2009.

[6] Douglas W Diamond and Raghuram G Rajan. Fear of fire sales, illiquidity seeking, and credit freezes. *The Quarterly Journal of Economics*, 126(2):557–591, 2011.

[7] Andrew Ellul, Chotibhak Jotikasthira, Christian T Lundblad, and Yihui Wang. Mark-to-market accounting and systemic risk: evidence from the insurance industry. *Economic Policy*, 29(78):297–341, 2014.

[8] Andrew Ellul, Chotibhak Jotikasthira, Christian T Lundblad, and Yihui Wang. Is historical cost accounting a panacea? market stress, incentive distortions, and gains trading. *The Journal of Finance*, 70(6):2489–2538, 2015.

[9] Joanne Horton and Richard Macve. The development of life assurance accounting and regulation in the uk: reflections on recent proposals for accounting change. *Accounting, Business Financial History*, 4(2):296–320, 1994.

[10] Michael C Jensen and William H Meckling. Theory of the firm: Managerial behavior, agency costs and ownership structure. *Journal of financial economics*, 3(4):305–360, 1976.

[11] Tong Lu, Haresh Sapra, and Ajay Subramanian. Agency conflicts, prudential regulation, and marking to market. *Chicago Booth Research Paper No. 11-05.*

[12] Guillaume Plantin, Haresh Sapra, and Hyun Song Shin. Marking-to-market: Panacea or pandora's box? *Journal of accounting research*, 46(2):435–460, 2008.

[13] United States Government Accountability Office. Insurance Markets: Impacts of and regulatory response to the 2007-2009 financial crisis. 2013.

[14] Lewis Webber and Rohan Churm. Decomposing corporate bond spreads. *Bank of England Quarterly Bulletin*, 47(4):533–541, 2016.

A. Figures

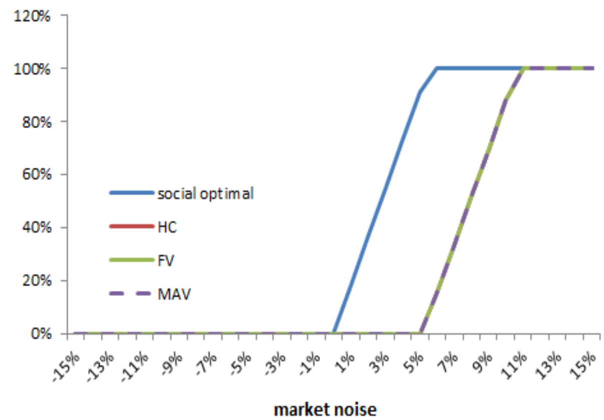


Figure 3. Equilibrium sales under the baseline calibration
The figure plots the equilibrium fraction of sales s^{HC}, given by (III.11), as a function of market noise expressed as a percentage of the fundamental value of the asset.

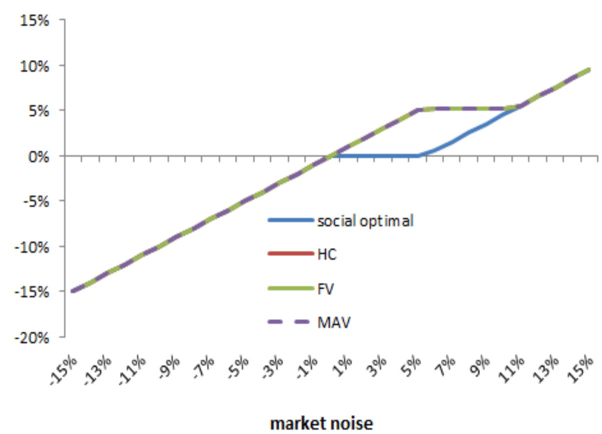


Figure 4. Market prices under the baseline calibration
The figure plots the market prices that prevails after insurers sell, v_1^{HC} given by (III.12), as a function of market noise expressed as a percentage of the fundamental value of the asset.

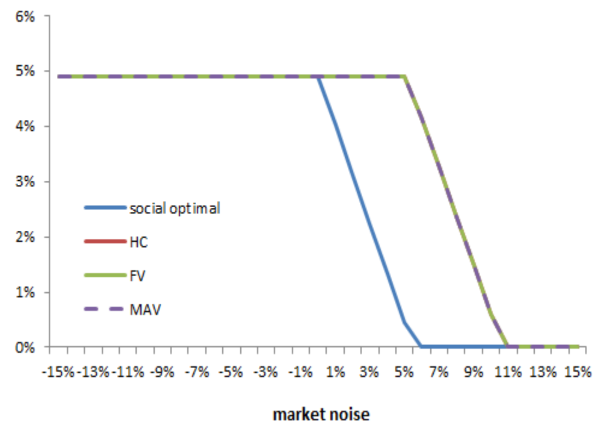


Figure 5. Policyholders losses under the baseline calibration
The figure plots the expected policyholders losses, when the three regimes coincide, as a function of market noise expressed as a percentage of the fundamental value of the asset.

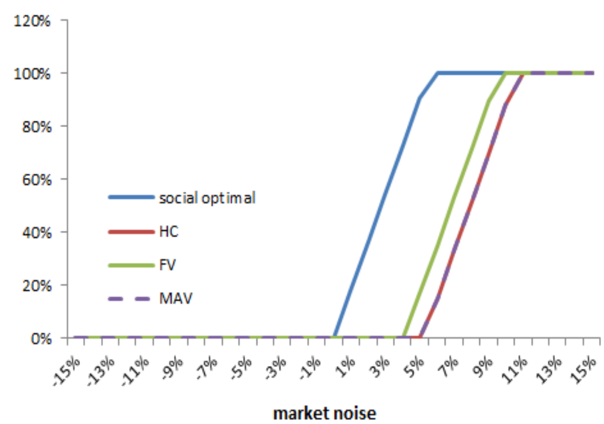


Figure 6. Equilibrium sales when $c^{MAV} < c < c^{FV}$ and $\rho < X(1 - z_1)$
The figure plots the equilibrium fraction of sales, when $c^{MAV} < c < c^{FV}$ and $\rho < X(1 - z_1)$, as a function of market noise expressed as a percentage of the fundamental value of the asset.

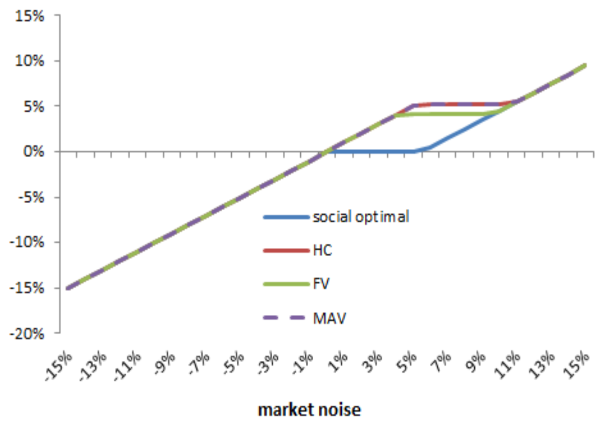


Figure 7. Market prices when $c^{MAV} < c < c^{FV}$ and $\rho < X(1 - z_1)$
The figure plots the market prices that prevails after insurers sell, when $c^{MAV} < c < c^{FV}$ and $\rho < X(1 - z_1)$, as a function of market noise expressed as a percentage of the fundamental value of the asset.

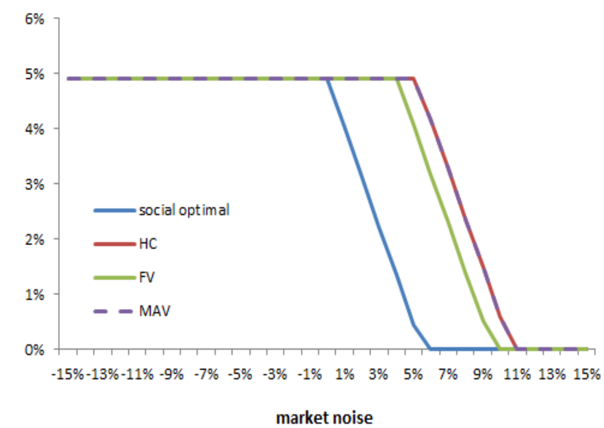


Figure 8. Policyholders losses when $c^{MAV} < c < c^{FV}$ and $\rho < X(1 - z_1)$
The figure plots the expected policyholders losses, when $c^{MAV} < c < c^{FV}$ and $\rho < X(1 - z_1)$, as a function of market noise expressed as a percentage of the fundamental value of the asset.

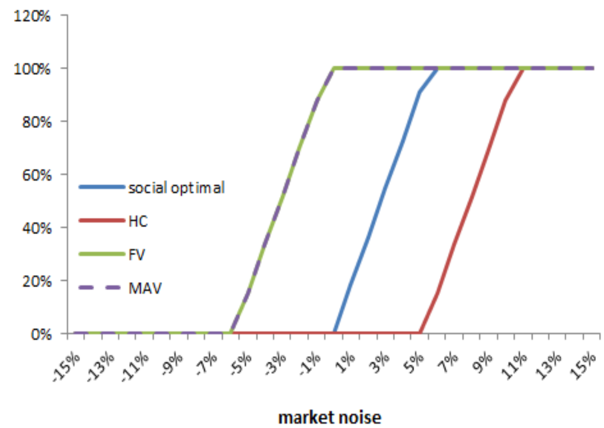


Figure 9. Equilibrium sales when $c < c^{MAV}$ and $\rho > X(1 - z_1)$
The figure plots the equilibrium fraction of sales, when $c < c^{MAV}$ and $\rho > X(1 - z_1)$, as a function of market noise expressed as a percentage of the fundamental value of the asset.

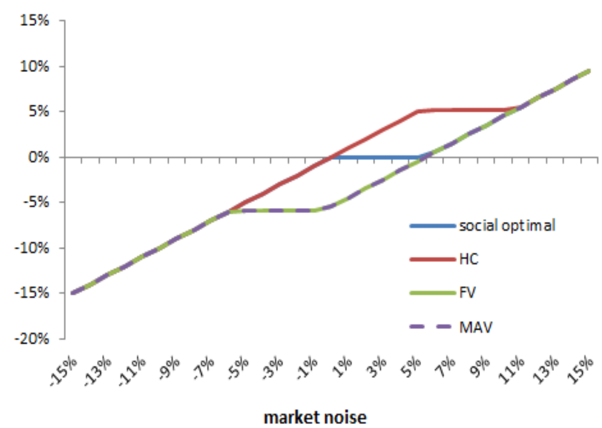


Figure 10. Market prices when $c < c^{MAV}$ and $\rho > X(1 - z_1)$
The figure plots the market prices that prevails after insurers sell, when $c < c^{MAV}$ and $\rho > X(1 - z_1)$, as a function of market noise expressed as a percentage of the fundamental value of the asset.

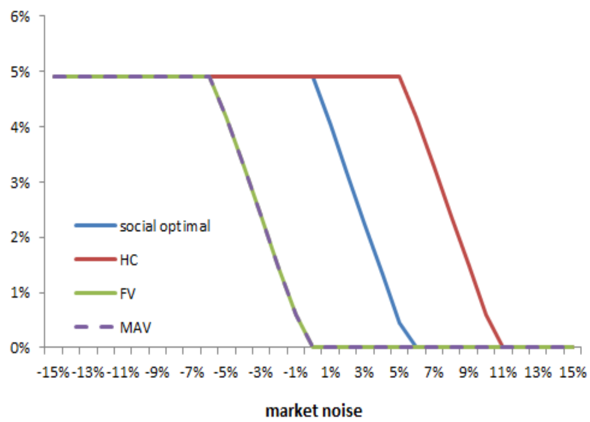


Figure 11. Policyholders losses when $c < c^{MAV}$ and $\rho > X(1 - z_1)$

The figure plots the expected policyholders losses, when $c < c^{MAV}$ and $\rho > X(1 - z_1)$, as a function of market noise expressed as a percentage of the fundamental value of the asset.

B. Proofs

A. Derivation of the solvency constraint

Below, we explain why the solvency requirement at $t = 0$ can be expressed in the form of (II.2). Define insurers' expected liabilities at time $t = 0$ as $L_0 = D_1 + D_2$. In practice insurance companies must have enough equity to cover:

$$E_0 \geq \Delta A_0 + \Delta L_0, \tag{B.1}$$

where ΔA_0 and ΔL_0 represent the change in assets and liabilities following a one-in-200-year shock. In our model we assume liabilities and interest rates to be constant, therefore it follows that $\Delta L_0 = 0$. On the other hand, assets are risky and therefore subject to changes in value in periods of stress. In particular we can write the asset value under stress as:

$$A_0^{stressed} = A_0 b + A_0(1 - b)c$$

where $1 - c$ is the percentage decline in the value of the risky asset that the regulator expects under stress. Note that, consistent with the rest of the model, we consider the risk-free asset constant and not affected by the shock. We then have that $\Delta A_0 = A_0(1 - b)(1 - c)$ and we can rewrite (B.1) as:

$$\begin{aligned}
&A_0 - (D_1 + D_2) \geq A_0(1 - b - (1 - b)c) \\
&1 - \frac{D_1 + D_2}{A_0} \geq 1 - b - (1 - b)c \\
&\frac{D_1 + D_2}{A_0(b + (1 - b)c)} \leq 1,
\end{aligned}$$

which is exactly (II.2). Analogously, we can derive the solvency constraint at $t = 1$ (III.3), starting from the following constraint:

$$V(A_1) - D_2 \geq \Delta V(A_1) + \Delta D_2 = \Delta V(A_1)$$

where we are still assuming no variation for the value of liabilities $\Delta D_2 = 0$. We indicate with $V(A_1)$ the value of the assets at time $t = 1$, where $A_1 = A_0(1 - b)$, which will depend on the valuation regime adopted. Using that $\Delta V(A_1) = V(A_1)(1 - c)$ we can re-write the above constraint as:

$$\frac{D_2}{cV(A_0(1 - b))} \leq 1$$

which is exactly (III.3).

B. Proof of Proposition (III.4): Fair Value Equilibrium

Under the fair value regime the expected payoff of the insurer that hold the assets, given that a fraction s sells, is:

$$z_1(R^H - X) - \rho * I(s > \bar{s}^{FV}) \tag{B.2}$$

where $\bar{s}^{FV}(c)$ is given by (III.14). Note that there are two extreme cases:

1. $\bar{s}^{FV} < 0$, which is equivalent to $c < \frac{X}{z_1 R + \varepsilon}$ or $\varepsilon < \frac{X}{c} - z_1 R$. In this case the constraint is always binding, even when there are no sales. This is the case when the market is very pessimistic, the fundamental value is very low, or the constraint is set very tight.

2. $\bar{s}^{FV} > 1$, which is equivalent to $c > \frac{X}{z_1 R + \varepsilon - \gamma}$. Then the constraint is never binding, not even when everyone sells. This is because the market price is always high enough (because either the market is very optimistic, higly liquid or the fundamental value is very high), such that even if everyone sells the constraint does not bind.

Define

$$c^{FV} \equiv \frac{X}{z_1 R + (1 - z_1) X}. \tag{B.3}$$

Then for $c \geq c^{FV}$ if a fraction $\pi^{HC} = \frac{1}{\gamma}[\varepsilon - X(1 - z_1)]$ sells the remaining fraction $1 - \pi^{HC}$ of insurers holding the asset satisfy the constraint. On the other hand when $c < c^{FV}$, the constraint is no longer satisfied and π^{HC} cannot be an equilibrium. In the rest of the proof we then analyse these two cases separetely.

Case 1: $c \geq c^{FV}$

i) When $\varepsilon < X(1 - z_1)$ no insurer sells under the historical cost regime and this remains the solution also under the fair regime, i.e. $s^{FV} = s^{HC} = 0$, as long as $\bar{s}^{FV} > 0$, that is $\varepsilon > \frac{X}{c} - z_1 R$. No insurer sells for this range of parameters because the market is unfavourable and the constraint is not breached as long as nobody sells. Note that the value of the penalty ρ is irrelevant in this case as nobody breaches the constraint.

ii) When $\varepsilon < X(1 - z_1)$ and $\varepsilon < \frac{X}{c} - z_1 R$, $s^{FV} = 0$ is still a solution if and only if the payoff from holding is greater than the payoff from selling, i.e.:

$$z_1(R - X) - \rho > [z_1 R + \varepsilon - X]^+$$

This is because $\bar{s}^{FV} \leq 0$ hence the solvency constraint is violated even if no insurer sells. Then $s = 0$ is a solution if the penalty is low enough $\rho \leq z_1(R - X)$ so that the payoff from holding remains positive and the market price is low due to over- relatively pessimistic $\varepsilon < X(1 - z_1) - \rho$. When $\rho \leq z_1(R - X)$ and $\varepsilon > X(1 - z_1) - \rho$ the aggregate sales is determined by the indifference condition of the last seller given that, by holding, it will breach the FV constraint. Thus, the equilibrium fraction of sellers π^{FV} is such that $z_1(R - X) - \rho = v_1(\pi^{FV}) - X$, from which we obtain:

$$\pi^{FV} = \frac{1}{\gamma}[\varepsilon - X(1 - z_1) + \rho] = \pi^{HC} + \frac{\rho}{\gamma}$$

Thus, if $c \geq c^{FV}$ and $\rho \leq z_1(R - X)$, but both $\varepsilon < X(1 - z_1)$ and $\varepsilon < \frac{X}{c} - z_1 R$, the equilibrium fraction of sales is given by (III.17).

While if $c \geq c^{FV}$, $\varepsilon < X(1 - z_1)$ and $\varepsilon < \frac{X}{c} - z_1 R$, but $\rho > z_1(R - X)$ everyone sells and the solution is $s^{FV} = 1$. This is because the penalty is so high that the payoff from holding becomes negative.

iii) When $X(1-z_1) \leq \varepsilon \leq \gamma + X(1-z_1)$ the equilibrium fraction of sales under the historical cost regime is π^{HC}. Because $c > c^{FV}$ we have that $\bar{s}^{FV} > \pi^{HC}$. Thus, this is still the equilibrium under the fair value regime since no insurer breaches the solvency constraint when π^{HC} sells. In fact there is no incentive for more than π^{HC} insurers to sell, given that their selling decision is determined by the condition (III.10). Thus, $s^{FV} = s^{HC} = \pi^{HC}$ in the parameter range $X(1-z_1) \leq \varepsilon \leq \gamma + X(1-z_1)$, regardless of the value of ρ.

iv) When $\varepsilon > \gamma + X(1-z_1)$ all insurers sell under the historical cost regime i.e. $s^{HC} = 1$. This is still a feasible solution, since $c \geq c^{FV}$ implies that $\bar{s}^{FV} > 1$ in this parameter range. Therefore the constraint is satisfied even when everyone sells and there is no incentive to deviate from the solution $s^{FV} = s^{HC} = 1$ regardless of the value of ρ.

To summarise, when $c \geq c^{FV}$ the equilibrium sales under fair value coincides with the one under historical cost $s^{FV} = s^{HC}$ when $\varepsilon > \frac{X}{c} - z_1 R$. Instead when $\varepsilon < \frac{X}{c} - z_1 R$ the equilibrium is $s^{FV} = 1$ if $\rho > z_1(R-X)$ and (III.17) if $\rho \leq z_1(R-X)$.

Case 2: $c < c^{FV}$

Consider now the case where the regulator sets the solvency requirement, such that $c < c^{FV}$. Under this constraint, if a fraction π^{HC} sells, then the remaining insurers will breach the solvency constraint regarless of whether they sell or hold the assets. In this case, (III.10) no longer pins down the indifference condition between selling and holding, and thus π^{HC} is no longer a feasible equilibrium.

i) When $\varepsilon < X(1-z_1)$ the solution is the same as in case 1 for $c \geq c^{FV}$. Thus $s^{FV} = s^{HC} = 0$, as long as $\bar{s}^{FV} > 0 \Leftrightarrow \varepsilon > \frac{X}{c} - z_1 R$.

ii) When $\varepsilon < X(1-z_1)$ and $\varepsilon < \frac{X}{c} - z_1 R$, then the solution is given by (III.17) when $\rho \leq z_1(R-X)$, and $s^{FV} = 1$ when $\rho > z_1(R-X)$.

iii) When $X(1-z_1) \leq \varepsilon \leq \gamma + X(1-z_1)$, $c < c^{FV}$ implies that $\bar{s}^{FV} < \pi^{HC}$. Thus, insurers that continue to hold the assets breach the solvency constraint when a fraction π^{HC} sells. In this case, all insurers sell and $s^{FV} = 1$ if the penalty is larger than $\rho > z_1(R-X)$, while $s^{FV} = \min(\pi^{FV}, 1)$ if $\rho \leq z_1(R-X)$. We verify that $\pi^{FV} > \bar{s}^{FV}$ that is $c < \frac{X}{z_1 R + (1-z_1)X - \rho}$, which always holds when $c < c^{FV}$. We also verify that the payoff from selling is non-negative, that is $z_1 R + \varepsilon - \gamma \pi^{FV} > 0$, which is always true when $\rho \leq z_1(R-X)$.

iv) When $\varepsilon > \gamma + X(1-z_1)$ everyone sells $s^{FV} = 1$ and remain solvent as long as $\frac{X}{z_1 R + \varepsilon - \gamma} < c$. On the other hand if $c < \frac{X}{z_1 R + \varepsilon - \gamma}$ when all insurers sell the solvency constraint is breached. Then if $\rho \leq z_1(R-X)$, the payoff from holding would still be positive and $s = 1$ arises when the market is realatively optimistic such that all insurers can realise a higher payoff by selling rather than holding. That is the case when $z_1(R-X) - \rho < v_1(1) - X$, which can be re-organised as:

$$\varepsilon > \gamma + X(1-z_1) - \rho$$

Finally, if $\rho > z_1(R-X)$ the payoff from holding becomes negative and everyone sells for any $\varepsilon > \gamma + X(1-z_1)$.

To summarise, if $c < c^{FV}$ and $\rho \geq z_1(R - X)$, $s^{FV} = 1$, while if $\rho \leq z_1(R - X)$, the solution is given by (III.17).

C. Proof of Proposition (III.5)

If $z_1 R > X$, then it follows that $X(1 - z_1) < z_1(R - X)$. Comparing (III.17) with (III.6) and (III.11) when $c \leq c^{FV}$:

1) $s^{FV} = s^* \; \forall \varepsilon$, when $\rho = X(1 - z_1)$.

2) $s^{HC} < s^* < s^{FV}$ for $X(1 - z_1) - \rho \leq \varepsilon \leq \gamma$ if $\rho > X(1 - z_1)$; and

3-i) $s^{HC} < s^{FV} < s^*$ for $0 < \varepsilon < \gamma + X(1 - z_1)$ if $\rho < X(1 - z_1)$.

3-ii) follows from proofs of Propositions (III.2) and (III.4).

D. Proof of Proposition (III.6): Policyholders losses

First let us analyse the case when $\rho < z_1(R - X)$. In this case the losses under FV are given by (III.21). Then for $\varepsilon < X(1 - z_1) - \rho$ the losses are L^{Hold} under both HC and FV, and for $\varepsilon > \gamma + X(1 - z_1)$ are zero. Hence the losses under HC and FV differs only when $X(1 - z_1) - \rho < \varepsilon < \gamma + X(1 - z_1)$.

In particular, if $\gamma > \rho$ we have that:

- for $X(1 - z_1) - \rho < \varepsilon < X(1 - z_1)$, $L^{FV} = (1 - \pi^{FV})L^{Hold} < L^{HC} = L^{Hold}$
- for $X(1 - z_1) < \varepsilon < \gamma - \rho + X(1 - z_1)$, $L^{FV} = (1 - \pi^{FV})L^{Hold} < L^{HC} = (1 - \pi^{HC})L^{Hold}$
- for $\gamma - \rho + X(1 - z_1) < \varepsilon < \gamma + X(1 - z_1)$ then $L^{FV} = L^{Sell}(1) = 0 < L^{HC} = (1 - \pi^{HC})L^{Hold}$

Note that in this last case we have that $L^{Sell}(1) = 0$ since $\varepsilon > \gamma - \rho + X(1 - z_1)$ implies that $\varepsilon > X - z_1 R + \frac{\gamma}{2}$, using that $\rho < z_1(R - X)$.

If instead $\gamma < \rho$ we have that:

- for $X(1 - z_1) - \rho < \varepsilon < \gamma - \rho + X(1 - z_1)$, $L^{FV} = (1 - \pi^{FV})L^{Hold} < L^{HC} = L^{Hold}$
- for $\gamma - \rho + X(1 - z_1) < \varepsilon < X(1 - z_1)$, $L^{FV} = L^{Sell}(1) = 0 < L^{HC} = L^{Hold}$
- for $X(1 - z_1) < \varepsilon < \gamma + X(1 - z_1)$, $L^{FV} = L^{Sell}(1) = 0 < L^{HC} = (1 - \pi^{HC})L^{Hold}$

Note that in the last two cases, we have that $L^{Sell}(1) = 0$ because $\varepsilon > \gamma - \rho + X(1 - z_1)$ implies that $\varepsilon > X - z_1 R + \frac{\gamma}{2}$, using that $\rho < z_1(R - X)$.

We now turn to the case when $\rho > z_1(R - X)$. In this case the losses under FV are $L^{Sell}(1)$, where:

$$L^{Sell}(1) = \begin{cases} D_2 - A_0(1 - b)(z_1 R + \varepsilon - \frac{\gamma}{2}), \text{if } \varepsilon < X - z_1 R + \frac{\gamma}{2} \\ 0, \text{if } \varepsilon > X - z_1 R + \frac{\gamma}{2} \end{cases} \tag{B.4}$$

Therefore, for $\varepsilon > X - z_1 R + \frac{\gamma}{2}$, $L^{FV} \leq L^{HC}$. For $\varepsilon < X - z_1 R + \frac{\gamma}{2}$, we can have that the losses under FV can be higher or lower than under HC depending on ε. In particular if $\frac{\gamma}{2} < z_1(R - X)$ we have that, for $\varepsilon < X - z_1 R + \frac{\gamma}{2}$, $L^{HC} = L^{Hold}$ is lower than $L^{FV} = L^{Sell}(1)$ if:

$$D_2 - A_0(1 - b)(z_1 R + \varepsilon - \frac{\gamma}{2}) > D_2(1 - z_1)$$

that is

$$\varepsilon < \frac{\gamma}{2} - z_1(R-X)$$

If $\frac{\gamma}{2} > z_1(R-X)$ we have that for $X(1-z_1) < \varepsilon < X - z_1R + \frac{\gamma}{2}$, then $L^{FV} = L^{Sell}(1) > L^{HC} = (1-\pi^{HC})L^{Hold}$ if

$$\begin{aligned}
&D_2 - A_0(1-b)(z_1R + \varepsilon - \frac{\gamma}{2}) > (1-\pi^{HC})D_2(1-z_1)\\
&D_2 - A_0(1-b)(z_1R + \varepsilon - \frac{\gamma}{2}) > (1-\frac{1}{\gamma}(\varepsilon - X(1-z_1)))D_2(1-z_1)\\
&-z_1R - \varepsilon + \frac{\gamma}{2} > -z_1X - \frac{1}{\gamma}(1-z_1)X(\varepsilon - X(1-z_1))
\end{aligned}$$

that is

$$X(1-z_1) < \varepsilon < \frac{1}{1-\frac{(1-z_1)X}{\gamma}}(\frac{\gamma}{2} - z_1(R-X) - \frac{1}{\gamma}(X(1-z_1))^2)$$

. For $\varepsilon < X(1-z_1)$ we have that $L^{FV} = L^{Sell}(1) < L^{HC} = L^{Hold}$ if $\varepsilon < \frac{\gamma}{2} - z_1(R-X)$ as we have shown above.

E. Proof of Proposition (III.7): Market Adjusted Valuation Equilibrium

Under the market adjusted valuation the expected payoff of the insurer that hold the assets, given that a fraction s sells, is:

$$z_1(R-X) - \rho * I(s > \bar{s}^{MAV})$$

where $\bar{s}^{MAV}$ is defined in (III.25) such that once $\bar{s}^{MAV}$ or more insurers sell their assets, the remaining insurers that hold the asset will breach the solvency constraint c. Note that:

1. $\bar{s}^{MAV} < 0$ if and only if $\varepsilon < \varepsilon^{MAV} \equiv \frac{1}{1-\theta}[\frac{X}{c} - (1-\theta)z_1R - \theta R]$. In this case the constraint is always binding even when there are no sales.
2. $\bar{s}^{MAV} > 1$ if and only if $\varepsilon > \varepsilon^{MAV} + \frac{1}{1-\theta}\gamma$ or equivalently, when $\epsilon > \gamma - z_1R$, if $c > \frac{X}{\theta R + (1-\theta)(z_1R + \varepsilon - \gamma))}$. In this case the constraint is never binding, not even when everyone sells.

Define

$$c^{MAV} \equiv \frac{X}{\theta R + (1-\theta)\left[z_1R + (1-z_1)X\right]}$$

Then for $c < c^{MAV}$ if a fraction π^{HC} sells the constraints binds on the remaining fraction $1-\pi^{HC}$ who hold. The rest of the proof will follow the same lines of the proof of the equilibrium under the fair value regime.

Case 1: $c \geq c^{MAV}$

i) When $\varepsilon < X(1-z_1)$, no insurers sell under the historical cost regime. This remains the solution also under MAV, i.e. $s^{MAV} = s^{HC} = 0$, as long as $\bar{s}^{MAV} > 0 \Leftrightarrow \varepsilon > \varepsilon^{MAV}$.

ii) When $\varepsilon < X(1-z_1)$ and $\varepsilon \leq \varepsilon^{MAV}$, we have that $\bar{s}^{MAV} \leq 0$ so that the solvency constraint is violated even if no insurer sells. In this case, $s^{MAV} = 0$ is still a solution if and only if the payoff from holding is greater than the payoff from selling, i.e.:

$$z_1(R-X) - \rho > [z_1 R + \varepsilon - X]^+$$

Then, analogously to Case 1i) of proposition (III.4), if the penalty is low enough $\rho < z_1(R-X)$ everyone holds ($s^{MAV} = 0$) if the market is relatively pessimistic $\varepsilon < X(1-z_1) - \rho$. While when $\rho < z_1(R-X)$ and $\varepsilon > X(1-z_1) - \rho$ the aggregate sales is determined by the indifference condition of the last seller given that, by holding, it will breach the MAV constraint, and the solution in this case is given by π^{FV}. Thus in this case the equilibrium fraction of sales is given by (III.17) if $\rho < z_1(R-X)$. On the other hand everyone sells and $s^{MAV} = 1$ when the penalty is large $\rho > z_1(R-X)$.

iii) When $X(1-z_1) \leq \varepsilon \leq \gamma + X(1-z_1)$ the equilibrium fraction of sales is π^{HC} as under the historical cost regime. This is because, since $c \geq c^{MAV}$, the remaining fraction $1-\pi^{HC}$ that holds the assets does not breach the constraint.

iv) When $\varepsilon > \gamma + X(1-z_1)$ everyone sells under the historical cost regime $s^{HC} = 1$, and this is still a feasible solution when $c > c^{MAV}$, since the MAV constraint is satisfied in this parameter range.

Case 2: $c < c^{MAV}$

Suppose now that the regulator sets the fair value solvency requirement, such that $c \leq c^{MAV}$. Under this regulatory solvency constraint, if π^{HC} sells, then the remaining insurers will breach the solvency constraint. In this case, (III.10) no longer pins down the indifference condition between selling and holding, and thus π^{HC} is no longer a feasible equilibrium.

i) When $\varepsilon < X(1-z_1)$ everyone holds and the solution is $s^{MAV} = s^{HC} = 0$, as long as $\bar{s}^{MAV} > 0$ $\Leftrightarrow \varepsilon > \varepsilon^{MAV}$.

ii) When both $\varepsilon < X(1-z_1)$ and $\varepsilon < \varepsilon^{MAV}$, then analogously to the fair value regime, the solution is (III.17) when $\rho \leq z_1(R-X)$, and $s^{MAV} = 1$ when $\rho > z_1(R-X)$.

iii) When $X(1-z_1) \leq \varepsilon \leq \gamma + X(1-z_1)$, $c \leq c^{MAV}$ implies that $\bar{s}^{MAV} \leq \pi^{HC}$. Thus, insurers that continue to hold the assets breaches the MAV solvency constraint when π^{HC} sells. In this case, all insurers sell and $s^{MAV} = 1$ if $\rho > z_1(R-X)$, while $s^{MAV} = \pi^{FV}$ if $\rho \leq z_1(R-X)$. We verify that $\pi^{FV} > \bar{s}^{MAV}$ as long as

$$c < \frac{X}{\theta R + (1-\theta)\left[z_1 R + (1-z_1)X - \rho\right]},$$

which always holds for $c < c^{MAV}$. We also verify that the payoff from selling is non-negative, i.e. $v_1(\pi^{FV}) > 0$, as long as $\rho \leq z_1(R-X)$.

iv) When $\varepsilon > \gamma + X(1-z_1)$ everyone selles $s^{MAV} = 1$ as long as

$$\frac{X}{\theta R + (1-\theta)(z_1 R + \varepsilon - \gamma)} < c \leq c^{MAV}.$$

On the other hand if $c < \frac{X}{\theta R+(1-\theta)(z_1 R+\varepsilon-\gamma)}$ and $\rho \leq z_1(R-X)$, $s=1$ arises when the market is very optimistic such that all insurers can realise a higher payoff by selling than holding and breaching the constraint: $z_1(R-X)-\rho < v_1(1)-X$, which can be re-organised as:

$$\varepsilon > \gamma + X\left(1-z_1\right) - \rho$$

Finally, if $\rho > z_1(R-X)$ then $s^{MAV}=1$

To summarise, if $c < c^{MAV}$ and $\rho > z_1(R-X)$, $s^{MAV}=1$. If $c < c^{MAV}$ and$\rho \leq z_1(R-X)$, the solution is given by (III.17).

F. *Proof of Proposition (IV.5): Ex-ante asset allocation*

The proof is divided in three parts, where we show each of the three properties.

i) *We begin by showing the first property $w^{HC} > w^{FV}$.* That is insurers' expected payoff from the risky strategy, given a downgrade, is higher under historical cost than under fair value. This implies that $p^{HC} > p^{FV}$, and thus the ex-ante asset allocation is safer under fair value.

Note that we can rewrite w^{HC} as:

$$\begin{aligned}
w^{HC} \quad &= z_1(R-X) + \int_{(1-z_1)X}^{\gamma+X(1-z_1)} \left\{\pi^{HC}\left[\varepsilon - \gamma\left(\frac{\pi^{HC}}{2}\right) - X(1-z_1)\right]\right\} f\left(\varepsilon\right) d\varepsilon \\
&+ \int_{\gamma+X(1-z_1)}^{\varepsilon^H} [\varepsilon - \frac{\gamma}{2} - X(1-z_1)] f\left(\varepsilon\right) d\varepsilon \\
&= z_1(R-X) + \int_{(1-z_1)X}^{\gamma+X(1-z_1)} \left\{\pi^{HC}\frac{1}{2}(\varepsilon - X(1-z_1))\right\} f\left(\varepsilon\right) d\varepsilon \\
&+ \int_{\gamma+X(1-z_1)}^{\varepsilon^H} [\varepsilon - \frac{\gamma}{2} - X(1-z_1)] f\left(\varepsilon\right) d\varepsilon
\end{aligned}$$

When $\rho \leq z_1(R-X)$, w^{FV} is given by:

$$\begin{aligned}
w^{FV} \quad &= z_1(R-X) + \int_{(1-z_1)X-\rho}^{\gamma+(1-z_1)X-\rho} \left\{\pi^{FV}\left[\varepsilon - \gamma\left(\frac{\pi^{FV}}{2}\right) - X(1-z_1)\right]\right\} f\left(\varepsilon\right) d\varepsilon \\
&-\rho\int_{(1-z_1)X-\rho}^{\gamma-(1-z_1)X-\rho} (1-\pi^{FV}) f\left(\varepsilon\right) d\varepsilon + \int_{\gamma+(1-z_1)X-\rho}^{\varepsilon^H} \left[\varepsilon - \frac{\gamma}{2} - X(1-z_1)\right] f\left(\varepsilon\right) d\varepsilon \\
&= z_1(R-X) + \int_{(1-z_1)X-\rho}^{\gamma+(1-z_1)X-\rho} \left\{\pi^{FV}\frac{1}{2}(\varepsilon - \rho - X(1-z_1))\right\} f\left(\varepsilon\right) d\varepsilon \\
&-\rho\int_{(1-z_1)X-\rho}^{\gamma-(1-z_1)X-\rho} (1-\pi^{FV}) f\left(\varepsilon\right) d\varepsilon + \int_{\gamma+(1-z_1)X-\rho}^{\varepsilon^H} \left[\varepsilon - \frac{\gamma}{2} - X(1-z_1)\right] f\left(\varepsilon\right) d\varepsilon
\end{aligned}$$

Therefore we have that the difference between the two expected payoffs is given by:

$$
\begin{aligned}
w^{HC} - w^{FV} \qquad &= \int_{X(1-z_1)}^{\gamma+X(1-z_1)} \left\{ \pi^{HC} \frac{1}{2} (\varepsilon - X(1-z_1)) \right\} f(\varepsilon) + \int_{\gamma+X(1-z_1)}^{\varepsilon^H} [\varepsilon - \frac{\gamma}{2} - X(1-z_1)] f(\varepsilon)\, d\varepsilon \\
&- \int_{(1-z_1)X-\rho}^{\gamma+(1-z_1)X-\rho} \left\{ \pi^{FV} \frac{1}{2} (\varepsilon - \rho - X(1-z_1)) \right\} f(\varepsilon)\, d\varepsilon \\
&+ \rho \int_{(1-z_1)X-\rho}^{\gamma-(1-z_1)X-\rho} (1-\pi^{FV}) f(\varepsilon)\, d\varepsilon - \int_{\gamma+(1-z_1)X-\rho}^{\varepsilon^H} \left[\varepsilon - \frac{\gamma}{2} - X(1-z_1) \right] f(\varepsilon)\, d\varepsilon.
\end{aligned}
$$

Note that

$$
\int_{\gamma+X(1-z_1)}^{\varepsilon^H} [\varepsilon - \frac{\gamma}{2} - X(1-z_1)] f(\varepsilon)\, d\varepsilon - \int_{\gamma+(1-z_1)X-\rho}^{\varepsilon^H} \left[\varepsilon - \frac{\gamma}{2} - X(1-z_1) \right] f(\varepsilon)\, d\varepsilon = \frac{1}{(\varepsilon^H - \varepsilon^L)} \frac{\rho}{2} [\rho - \gamma].
$$

Furthermore, the first term equals to:

$$
\int_{X(1-z_1)}^{\gamma+X(1-z_1)} \left\{ \pi^{HC} \frac{1}{2} (\varepsilon - X(1-z_1)) \right\} f(\varepsilon) = \frac{1}{(\varepsilon^H - \varepsilon^L)} \frac{\gamma^2}{6},
$$

and the remaining terms can be expressed as:

$$
- \int_{(1-z_1)X-\rho}^{\gamma+(1-z_1)X-\rho} (\pi^{FV} \frac{1}{2} (\varepsilon - \rho - X(1-z_1)) + \rho \pi^{FV}) f(\varepsilon)\, d\varepsilon = - \frac{1}{(\varepsilon^H - \varepsilon^L)} \frac{1}{6} \gamma^2.
$$

Then the above expression can be simplified to:

$$
w^{HC} - w^{FV} = \frac{\rho}{(\varepsilon^H - \varepsilon^L)} \left(\frac{\rho + \gamma}{2} \right),
$$

which is always positive. Therefore, when $\rho \leq z_1(R - X)$, we have proved that HC leads to a riskier asset allocation than FV.

We will now show that the same is true when $\rho > z_1(R - X)$. In this case the expected payoff under FV is given by:

$$
w^{FV} = z_1(R - X) + \int_{\varepsilon^L}^{\varepsilon^H} (\varepsilon - \frac{\gamma}{2} - X(1-z_1)) f(\varepsilon)\, d\varepsilon. \tag{B.5}
$$

Note that we can rewrite the payoff under HC as:

$$
w^{HC} = z_1(R - X) + \frac{1}{(\varepsilon^H - \varepsilon^L)} \frac{\gamma^2}{6} + \int_{\gamma+X(1-z_1)}^{\varepsilon^H} (\varepsilon - \frac{\gamma}{2} - X(1-z_1)) f(\varepsilon)\, d\varepsilon.
$$

Then

$$w^{HC} - w^{FV} = \frac{1}{(\varepsilon^H - \varepsilon^L)}\frac{\gamma^2}{6} + \frac{1}{(\varepsilon^H - \varepsilon^L)}(\frac{\gamma}{2}X(1-z_1) - \frac{\gamma}{2}\varepsilon^H + \frac{(X(1-z_1))^2}{2} - \varepsilon^L X(1-z_1) + \frac{(\varepsilon^H)^2}{2}) \geq 0$$

which is always positive since $\varepsilon^L < 0$. Therefore we have proved that also in this case HC leads to a riskier asset allocation than FV.

ii) *We now show the second property:* $w^{HC} + X > w^s$. This implies that under the historical cost regime insurers adopt riskier investment strategies than what socially optimal.

Note that we can write

$$\begin{aligned} w^{HC} + X = z_1 R + X(1-z_1) + \textstyle\int_{(1-z_1)X}^{\gamma+X(1-z_1)} \left\{\pi^{HC}\frac{1}{2}(\varepsilon - X(1-z_1))\right\} f(\varepsilon)\, d\varepsilon \\ + \textstyle\int_{\gamma+X(1-z_1)}^{\varepsilon^H} [\varepsilon - \frac{\gamma}{2} - X(1-z_1)] f(\varepsilon)\, d\varepsilon \end{aligned}$$

and

$$w^s = z_1 R + \textstyle\int_0^{\gamma} \frac{\varepsilon^2}{2\gamma} f(\varepsilon)\, d\varepsilon + \int_{\gamma}^{\varepsilon^H} (\varepsilon - \frac{\gamma}{2}) f(\varepsilon)\, d\varepsilon$$

Then, it is possible to show that:

$$\begin{aligned} w^{HC} + X - w^s &= X(1-z_1)\left[1 - \int_{X(1-z_1)}^{\gamma+X(1-z_1)} \frac{\pi^{HC}}{2} f(\varepsilon)\, d\varepsilon - \int_{\gamma+X(1-z_1)}^{\varepsilon^H} f(\varepsilon)\, d\varepsilon - \int_{X(1-z_1)}^{\gamma+X(1-z_1)} \frac{\varepsilon}{2\gamma} f(\varepsilon)\, d\varepsilon\right] \\ &+ \int_{X(1-z_1)}^{\gamma+X(1-z_1)} \frac{\epsilon^2}{2\gamma} f(\varepsilon)\, d\varepsilon - \int_0^{\gamma} \frac{\epsilon^2}{2\gamma} f(\varepsilon)\, d\varepsilon + \int_{\gamma+X(1-z_1)}^{\varepsilon^H} (\varepsilon - \frac{\gamma}{2}) f(\varepsilon)\, d\varepsilon - \int_{\gamma}^{\varepsilon^H} (\varepsilon - \frac{\gamma}{2}) f(\varepsilon)\, d\varepsilon \\ &= X(1-z_1)\left[1 + \frac{1}{\varepsilon^H - \varepsilon^L}\left(\frac{X(1-z_1)}{2} - \varepsilon^H + \gamma + X(1-z_1)\right)\right] \\ &+ \frac{1}{(\varepsilon^H - \varepsilon^L)2\gamma}\left[\frac{3\gamma^2 X(1-z_1) + 3\gamma(X(1-z_1))^2}{3}\right] + \frac{1}{2(\varepsilon^H - \varepsilon^L)}\left[\gamma X(1-z_1) - (\gamma + X(1-z_1))^2 + \gamma^2\right] \\ &= X(1-z_1)\left(\frac{\frac{3}{2}X(1-z_1) + \gamma - \varepsilon^L}{\varepsilon^H - \varepsilon^L}\right) \end{aligned}$$

which is always positive, since ε^L is negative. Therefore we have shown that $w^{HC} + X > w^s$.

iii) *Finally we want to understand when* $w^s > w^{FV} + X$. Comparing the expected payoff from the risky strategy under fair value to the socially optimal one, we will prove that, under some conditions, fair value induces a more prudent asset allocation that what socially optimal.

First consider the case when $\rho \leq z_1(R - X)$. In this case w^{FV} is given by:

$$\begin{aligned} w^{FV} &= z_1(R - X) + \int_{(1-z_1)X-\rho}^{\gamma+(1-z_1)X-\rho} \left\{\pi^{FV}\frac{1}{2}(\varepsilon - \rho - X(1-z_1))\right\} f(\varepsilon)\, d\varepsilon \\ &- \rho\int_{(1-z_1)X-\rho}^{\gamma-(1-z_1)X-\rho} (1 - \pi^{FV}) f(\varepsilon)\, d\varepsilon + \int_{\gamma+(1-z_1)X-\rho}^{\varepsilon^H} \left[\varepsilon - \frac{\gamma}{2} - X(1-z_1)\right] f(\varepsilon)\, d\varepsilon \end{aligned}$$

Then we have that:

$$w^s - w^{FV} - X = -X(1-z_1) + \int_0^{\gamma} \frac{\varepsilon^2}{2\gamma} f(\varepsilon)\, d\varepsilon + \int_{\gamma}^{\varepsilon^H} (\varepsilon - \frac{\gamma}{2}) f(\varepsilon)\, d\varepsilon - \int_{\gamma+X(1-z_1)-\rho}^{\varepsilon^H} (\varepsilon - \frac{\gamma}{2}) f(\varepsilon)\, d\varepsilon$$
$$+ \int_{X(1-z_1)-\rho}^{\gamma-\rho+X(1-z_1)} \frac{\pi^{FV}}{2} (X(1-z_1) - \rho - \varepsilon) f(\varepsilon)\, d\varepsilon + \rho \int_{(1-z_1)X-\rho}^{\gamma+(1+z_1)X-\rho} f(\varepsilon)\, d\varepsilon + \int_{\gamma-\rho+X(1-z_1)}^{\varepsilon^H} X(1-z_1) f(\varepsilon)\, d\varepsilon$$

The third and fourth terms can be simplified to:

$$\int_{\gamma}^{\varepsilon^H} (\varepsilon - \frac{\gamma}{2}) f(\varepsilon)\, d\varepsilon - \int_{\gamma+X(1-z_1)-\rho}^{\varepsilon^H} (\varepsilon - \frac{\gamma}{2}) f(\varepsilon)\, d\varepsilon = \frac{1}{2(\varepsilon^H - \varepsilon^L)} ((1-z_1)X - \rho)((1-z_1)X - \rho + \gamma).$$

The fifth term is equal to

$$\int_{X(1-z_1)-\rho}^{\gamma-\rho+X(1-z_1)} \frac{\pi^{FV}}{2} (X(1-z_1) - \rho - \varepsilon) f(\varepsilon)\, d\varepsilon = -\frac{1}{2\gamma} \int_{X(1-z_1)-\rho}^{\gamma-\rho+X(1-z_1)} (\rho + \varepsilon - X(1-z_1))^2 f(\varepsilon)\, d\varepsilon$$
$$= -\frac{1}{(\varepsilon^H - \varepsilon^L)} \frac{1}{6} \gamma^2.$$

Then we can simplify the above expression to:

$$w^s - w^{FV} - X = -X(1-z_1) + \frac{\gamma^2}{6(\varepsilon^H - \varepsilon^L)} + \frac{1}{2(\varepsilon^H - \varepsilon^L)} ((1-z_1)X - \rho)((1-z_1)X - \rho + \gamma)$$
$$= \frac{1}{(\varepsilon^H - \varepsilon^L)} X(1-z_1) \left(\varepsilon^L - \frac{1}{2} X(1-z_1) - \frac{\gamma}{2} \right) + \frac{1}{(\varepsilon^H - \varepsilon^L)} \frac{\rho}{2} (\rho + \gamma).$$

This expression is positive if (IV.5) holds:

$$\rho(\rho + \gamma) > X(1-z_1)[X(1-z_1) + \gamma - 2\varepsilon^L].$$

Note that this condition is never satisfied when $X(1-z_1) \geq \rho$, thus $w^s < w^{FV} + X$ and FV leads to a riskier asset allocation than socially optimal. Let us now analyze the case when $\rho > z_1(R-X)$. In this case $w^s > w^{FV} + X$ always holds, hence FV leads to an asset allocation which is too prudent, relative to the one that is socially optimal. When $\rho > z_1(R-X)$ all insurers sell in equilibrium after a downgrade, thus the expected payoff at $t = 0$ is given by (B.5). Then

$$w^s - w^{FV} - X = -X(1-z_1) + \int_0^{\gamma} \frac{\varepsilon^2}{2\gamma} f(\varepsilon)\, d\varepsilon + \int_{\gamma}^{\varepsilon_H} (\varepsilon - \frac{\gamma}{2}) f(\varepsilon)\, d\varepsilon - \int_{\varepsilon^L}^{\varepsilon^H} (\varepsilon - \frac{\gamma}{2} - X(1-z_1)) f(\varepsilon)\, d\varepsilon$$
$$= \frac{1}{(\varepsilon^H - \varepsilon^L)} \left(\frac{\gamma^2}{6} + \frac{1}{2} (\varepsilon^L)^2 - \frac{\gamma}{2} \varepsilon^L \right)$$

which is always positive since ε^L is negative. Therefore the result follows.

G. Proof of (V.1): ex-ante policyholders losses

First we calculate the expected losses under HC at $t=0$:

$$E_0[L_1^{HC}] = \int_{\varepsilon^L}^{X(1-z_1)} L^{Hold} f(\varepsilon)\, d\varepsilon + \int_{X(1-z_1)}^{\gamma+X(1-z_1)} (1-\pi^{HC}) L^{Hold} f(\varepsilon)\, d\varepsilon$$

$$= \int_{\varepsilon^L}^{X(1-z_1)} D_2(1-z_1) f(\varepsilon)\, d\varepsilon + \int_{X(1-z_1)}^{\gamma+X(1-z_1)} (1-\frac{\varepsilon}{\gamma}+\frac{X(1-z_1)}{\gamma}) D_2(1-z_1) f(\varepsilon)\, d\varepsilon$$

$$= D_2(1-z_1)\left[\frac{X(1-z_1)-\varepsilon^L}{\varepsilon^H-\varepsilon^L}+\frac{\gamma}{\varepsilon^H-\varepsilon^L}\left(1+\frac{X(1-z_1)}{\gamma}\right)-\frac{[\gamma+X(1-z_1)]^2-[X(1-z_1)]^2}{2\gamma(\varepsilon^H-\varepsilon^L)}\right]$$

$$= \frac{D_2(1-z_1)}{\varepsilon^H-\varepsilon^L}[X(1-z_1)-\varepsilon^L+\gamma+X(1-z_1)-\frac{\gamma}{2}-X(1-z_1)]$$

$$= \frac{D_2(1-z_1)}{\varepsilon^H-\varepsilon^L}[-\varepsilon^L+X(1-z_1)+\frac{\gamma}{2}]$$

Second we calculate the expected losses under FV for $c < c^{FV}$. In particular, if $\rho < z_1(R-X)$ the expected losses at $t=0$ under FV are given by:

$$E_0[L_1^{FV}] = \int_{\varepsilon^L}^{X(1-z_1)-\rho} L^{Hold} f(\varepsilon)\, d\varepsilon + \int_{X(1-z_1)-\rho}^{\gamma+X(1-z_1)-\rho} (1-\pi^{FV}) L^{Hold} f(\varepsilon)\, d\varepsilon$$

$$= \int_{\varepsilon^L}^{X(1-z_1)-\rho} D(1-d)d(1-z_1) f(\varepsilon)\, d\varepsilon$$

$$+ \int_{X(1-z_1)-\rho}^{\gamma+X(1-z_1)-\rho} (1-\frac{\varepsilon}{\gamma}+\frac{X(1-z_1)}{\gamma}-\frac{\rho}{\gamma}) D_2(1-z_1) f(\varepsilon)\, d\varepsilon$$

$$= \frac{D_2(1-z_1)}{\varepsilon^H-\varepsilon^L}\Big\{X(1-z_1)-\rho-\varepsilon^L+(\gamma+X(1-z_1)-\rho)$$

$$-\frac{(\gamma+X(1-z_1)-\rho)^2-(X(1-z_1)-\rho)^2}{2\gamma}\Big\}$$

$$\frac{D_2(1-z_1)}{\varepsilon^H-\varepsilon^L}\left[2X(1-z_1)-2\rho-\varepsilon^L+\gamma-\frac{\gamma}{2}-X(1-z_1)+\rho\right]$$

$$= \frac{D_2(1-z_1)}{\varepsilon^H-\varepsilon^L}\left[-\varepsilon^L+X(1-z_1)+\frac{\gamma}{2}-\rho\right]$$

Therefore $E_0[L_1^{FV}] < E_0[L_1^{HC}]$ in this case. Furthermore, since the ex-ante asset allocation under FV is more prudent than under HC we can state that:

$$L_0^{FV} \le L_0^{HC}.$$

When $\rho > z_1(R-X)$ under FV everyone sells in equilibrium, but losses for policyholders only arise when the sales proceeds are not enough to pay them back, i.e. $D_2 - A_0(1-b)(z_1R+\varepsilon-\frac{\gamma}{2}) > 0$, that is if the market is sufficiently pessimistic: $\varepsilon^H < X - z_1R + \frac{\gamma}{2}$. If this is the case, the expected losses at $t=0$ under FV are equal to:

$$
\begin{aligned}
L_0^{FV} &= \int_{\varepsilon^L}^{\varepsilon^H} L^{Sell}(1) f(\varepsilon)\, d\varepsilon \\
&= \int_{\varepsilon^L}^{X - z_1 R + \frac{\gamma}{2}} [D_2 - A_0(1-b)(z_1 R + \varepsilon - \frac{\gamma}{2})] f(\varepsilon)\, d\varepsilon \\
&= \frac{1}{\varepsilon^H - \varepsilon^L}\left(D_2 - A_0(1-b)(z_1 R - \frac{\gamma}{2})\right)(X - z_1 R + \frac{\gamma}{2} - \varepsilon^L) \\
&\quad - \frac{1}{\varepsilon^H - \varepsilon^L} A_0(1-b) \frac{(X - z_1 R + \frac{\gamma}{2})^2 - (\varepsilon^L)^2}{2} \\
&= \frac{D_2}{\varepsilon^H - \varepsilon^L}(X - z_1 R + \frac{\gamma}{2} - \varepsilon^L) \\
&\quad - \frac{A_0(1-b)}{\varepsilon^H - \varepsilon^L}\left\{(z_1 R - \frac{\gamma}{2})(X - z_1 R + \frac{\gamma}{2} - \varepsilon^L) + \frac{(X - z_1 R + \frac{\gamma}{2})^2 - (\varepsilon^L)^2}{2}\right\} \\
&= \frac{D_2}{\varepsilon^H - \varepsilon^L}(X - z_1 R + \frac{\gamma}{2} - \varepsilon^L) \\
&\quad - \frac{A_0(1-b)}{\varepsilon^H - \varepsilon^L}\left\{(z_1 R - \frac{\gamma}{2})\left(X - \varepsilon^L\right) - (z_1 R - \frac{\gamma}{2})^2 + \frac{X^2}{2} - X(z_1 R - \frac{\gamma}{2}) - \frac{1}{2}(z_1 R - \frac{\gamma}{2})^2 - \frac{(\varepsilon^L)^2}{2}\right\} \\
&= \frac{D_2}{\varepsilon^H - \varepsilon^L}(X - z_1 R + \frac{\gamma}{2} - \varepsilon^L) \\
&\quad - \frac{A_0(1-b)}{\varepsilon^H - \varepsilon^L}\left\{(z_1 R - \frac{\gamma}{2})\left(-\varepsilon^L\right) - \frac{3}{2}(z_1 R - \frac{\gamma}{2})^2 + \frac{X^2}{2} - \frac{(\varepsilon^L)^2}{2}\right\}
\end{aligned}
$$

Comparing this with the losses under HC, we have that $E_0[L_1^{FV}] \leq E_0[L_1^{HC}]$ if

$$\varepsilon^L \geq X(1 - z_1) + X - R + \frac{\gamma}{2}.$$

Therefore when this holds, we conclude that $L_0^{FV} \leq L_0^{HC}$. When this is not true, $E_0[L_1^{FV}] > E_0[L_1^{HC}]$. But because the probability of downgrade under FV $p \leq p^{FV}$, while under HC $p \leq p^{HC}$ with $p^{FV} \leq p^{HC}$, we cannot conclude that in general FV leads to large losses than HC. This is true when, both under FV and HC, $p \leq p^{FV}$.

Finally we want to compare the losses under the two regimes to the losses obtained under the socially optimal strategy. Note that $L^{Sell}(1) = 0$ for $\varepsilon > \gamma$ because $\gamma > X - z_1 R + \frac{\gamma}{2}$, since by assumption $X < z_1 R$. Therefore :

$$
\begin{aligned}
E_0[L_1^*] &= \int_{\varepsilon^L}^{0} L^{Hold} f(\varepsilon)\, d\varepsilon + \int_0^{\gamma} (1 - \pi^*) L^{Hold} f(\varepsilon)\, d\varepsilon \\
&= \int_{\varepsilon^L}^{0} D_2(1 - z_1) f(\varepsilon)\, d\varepsilon + \int_0^{\gamma} (1 - \frac{\varepsilon}{\gamma}) D_2(1 - z_1) f(\varepsilon)\, d\varepsilon \\
&= \frac{D_2(1 - z_1)}{\varepsilon^H - \varepsilon^L}\left[-\varepsilon^L + \gamma - \frac{\gamma^2}{2\gamma}\right] \\
&= \frac{D_2(1 - z_1)}{\varepsilon^H - \varepsilon^L}\left[-\varepsilon^L + \frac{\gamma}{2}\right]
\end{aligned}
$$

Then the expected losses under the socially optimal rule are:

$$L_0^* = pE_0[L_1^*] = p\frac{D_2(1-z_1)}{\varepsilon^H - \varepsilon^L}\left[-\varepsilon^L + \frac{\gamma}{2}\right]$$

Note that $E_0[L_1^*] < E_0[L_1^{HC}]$. Furthermore HC leads to a riskier ex-ante asset allocation ($p^* \leq p^{HC}$). Therefore HC leads to larger losses than what socially optimal, $L_0^{HC} > L_0^*$. Let us now compare the socially optimal losses to the losses under FV. When $\rho \leq X(1-z_1)$, FV leads to larger losses then the socially optimal rule. This is because $E_0[L_1^*] < E_0[L_1^{FV}]$ and the ex-ante asset allocation is riskier ($p^* < p^{FV}$). When $X(1-z_1) \leq \rho \leq z_1(R-X)$, policyholders losses under FV are lower than what socially optimal when (IV.5) holds. This is because, $E_0[L_1^*] > E_0[L_1^{FV}]$ and (IV.5) implies that $p^{FV} < p^*$. Thus $L_0^{FV} \leq L_0^*$. Finally when $\rho > z_1(R-X)$ we have that $E_0[L_1^{FV}] > E_0[L_1^*]$. However, the ex-ante asset allocation is safer under FV, because $p^{FV} < p^*$, thus the overall effect on policyholders is ambiguous.